25 points possible. **No aids (internet, other students, book, calculator, etc.) are permitted.** You do not need to simplify final answers, but **answers without supporting work will lose points for completeness and effort.**

1. **[8 points]** Consider the curve $y = 3\sqrt{x}$ on the interval $0 \le x \le 4$.

   a. Sketch a graph of this curve.
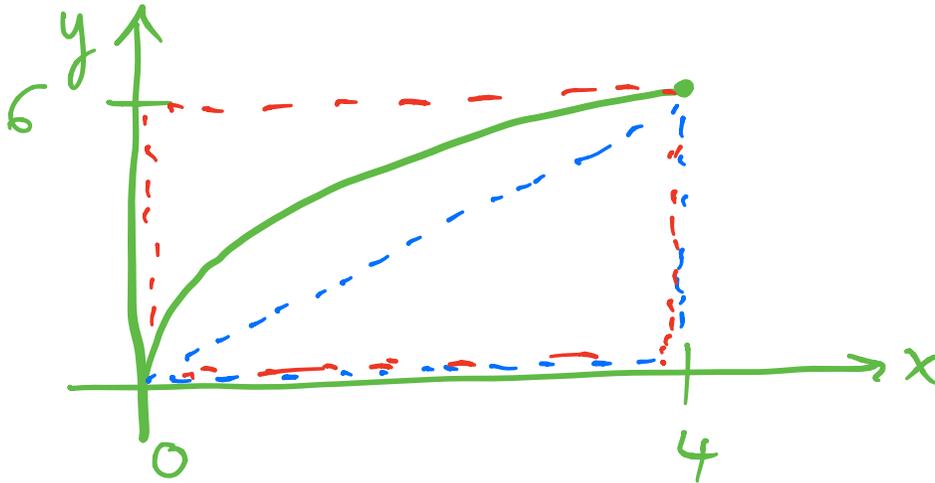
   b. Give a rough estimate of the area beneath the curve. **Explain** in words the approximation you are using, **and** sketch this estimate on top of your sketch above.

   $A = $ exact area

   blue triangle: $12 \le A$

   red rectangle: $A \le 24$

   c. Find the exact area.

   $$A = \int_0^4 3\sqrt{x}\,dx = 3\,\frac{x^{3/2}}{3/2}\Big]_0^4 = 2 \times^{3/2}\Big]_0^4$$
   $$= 2(8-0) = \boxed{16}$$

2. **[5 points]** Evaluate the integral.

   $$\int_0^1 x^\pi + e^x\,dx = \frac{x^{\pi+1}}{\pi+1} + e^x\Big]_0^1 = \left(\frac{1}{\pi+1} + e\right) - (0+1)$$
   $$= \boxed{\frac{1}{\pi+1} + e - 1}$$

3. [6 points] A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week.

    a. What does the integral $\int_0^7 n'(t)\,dt$ represent? (*Explain in a few words.*)

this is the net change in the number of bees between week 0 and week 7

    b. What does $100 + \int_0^7 n'(t)\,dt$ represent? (*Explain in a few words.*)

this is the total number of bees after 7 weeks

4. [6 points] Let $F(x) = \int_2^x \cos(\pi t^2)\,dt$. Find an equation of the tangent line to the curve $y = F(x)$ at the point where $x = 2$.

by the FTCI, $F'(x) = \cos(\pi x^2)$

So: $F(2) = \int_2^2 \cos(\pi t^2)\,dt = 0$

$F'(2) = \cos(\pi \cdot 2^2) = \cos(4\pi) = 1$

So $y - 0 = 1 \cdot (x - 2)$

$\boxed{y = x - 2}$