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Each problem is worth 2 points: 1 for work and 1 for the answer. Your work should be organized. Your answer should be in a $\boxed{\text{box}}$. This quiz is an excellent measure of readiness for Calculus II. You **should** be able to complete this in 30 minutes or less. To encourage you to pay attention to time, we have added blanks for that, too. If you use aids of any kind you are completely missing the point of this exercise.

Time Started: 1:10 PM **Time Started:**____________

Evaluate the indefinite and definite integrals.

1. [2 points] $\int 5\sin\left(\frac{\pi}{2}\theta\right)\,d\theta = 5\int \frac{2}{\pi}\sin(u)\,du = \frac{10}{\pi}(-\cos u) + C$

$= \boxed{\frac{-10}{\pi}\cos\left(\frac{\pi\theta}{2}\right) + C}$

let $u = \frac{\pi}{2}\theta$

$du = \frac{\pi}{2}d\theta$

$\frac{2}{\pi}du = d\theta$

2. [2 points] $\int 3x - e^{3x}\,dx = \boxed{\frac{3}{2}x^2 - \frac{1}{3}e^{3x} + C}$

method: guess-n-check

$\frac{d}{dx}\left[\frac{1}{3}e^{3x}\right] = 3e^{3x}\cdot\frac{1}{3} = e^{3x}$ ✓

3. [2 points] $\int \frac{1}{6-7t}dt = -\frac{1}{7}\int\frac{1}{u}du = -\frac{1}{7}\ln|u| + C$

let $u = 6-7t$

$du = -7dt$

$-\frac{1}{7}du = dt$

$= \boxed{-\frac{1}{7}\ln|6-7t| + C}$

4. [2 points] $\int \frac{ax}{\sqrt{1-ax^2}}dx = \int ax(1-ax^2)^{-1/2}dx = -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot u^{1/2}\cdot 2 + C$

let $u = 1-ax^2$

$du = -2ax\,dx$

$-\frac{1}{2}du = ax\,dx$

$= \boxed{-(1-ax)^{1/2} + C}$

5. [2 points] $\int_1^3 te^{t^2}\,dt$ $= \int_1^9 e^u \cdot \frac{1}{2}\,du = \frac{1}{2} e^u \Big]_1^9 = \boxed{\frac{1}{2}(e^9 - e)}$

let $u = t^2$

$du = 2t\,dt$

$\frac{1}{2}\,du = t\,dt$

If $t=1$, $u = 1^2 = 1$

If $t=3$, $u = 3^2 = 9$

6. [2 points] $\int_0^{\pi/4} \cos(2t)\,(1+\sin(2t))^2\,dt$ $= \frac{1}{2}\int_1^2 u^2\,du = \frac{1}{2}\cdot\frac{1}{3}\cdot u^3\Big]_1^2$

let $u = 1 + \sin(2t)$

$du = 2\cos(2t)\,dt$

$\frac{1}{2}\,du = \cos(2t)\,dt$

$= \frac{1}{6}\left(2^3 - 1^3\right) = \boxed{\frac{7}{6}}$

If $t=0$, $u=1$

If $t = \pi/2$, $u = 2$

7. [2 points] $\int_0^1 \frac{1+e^x}{x+e^x}\,dx$ $= \int_1^{1+e} \frac{du}{u} = \ln|u|\Big]_1^{1+e}$

let $u = x + e^x$

$du = (1+e^x)\,dx$

$= \ln(1+e) - \ln(1) = \boxed{\ln(1+e)}$

If $x=0$, $u=1$;

$x=1$, $u = 1+e$

8. [2 points] $\int \frac{dx}{1+16x^2}$ $= \int \frac{dx}{1+(4x)^2} = \frac{1}{4}\int \frac{du}{1+u^2}$

let $u = 4x$

$du = 4\,dx$

$\frac{1}{4}\,du = dx$

$= \frac{1}{4}\arctan u + C$

$= \boxed{\frac{1}{4}\arctan(4x) + C}$