Name: Solutions ______ / 25

Circle one: **Faudree (F01)** | **Bueler (F02)** | **VanSpronsen (UX1)**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[9 points]** Evaluate each limit below. Your answer for each should be either a real number, $+\infty$, $-\infty$, or DNE. Show your work to receive full credit.

a. $\lim_{x\to-3} \frac{x^2+2x-3}{x^2+5x+6}$ $= \cancel{\frac{9-6-3}{9-15+6} = \frac{0}{0}}$ ← factor + cancel!

$$= \lim_{x\to-3} \frac{(x+3)(x-1)}{(x+3)(x+2)} = \lim_{x\to-3} \frac{x-1}{x+2} = \frac{-3-1}{-3+2} = \frac{-4}{-1} = 4$$

b. $\lim_{x\to4} \frac{2-\sqrt{x}}{4x-x^2}$ $= \cancel{\frac{2-2}{16-16} = \frac{0}{0}}$ ← Do algebra.

$$= \lim_{x\to4} \frac{2-\sqrt{x}}{4x-x^2} \cdot \frac{(2+\sqrt{x})}{(2+\sqrt{x})} = \lim_{x\to4} \frac{4-x}{x(4-x)(2+\sqrt{x})} = \lim_{x\to4} \frac{1}{x(2+\sqrt{x})} = \frac{1}{4(2+2)} = \frac{1}{16}$$

c. $\lim_{h\to0^-} \frac{2h^2+14h}{|h|}$ $= \lim_{h\to0^-} \frac{2h(h+7)}{-h} = \lim_{h\to0^-} -2(h+7) = -14$

If $h<0$, then $|h| = -h$.

2. **[4 points]** Use the Intermediate Value Theorem to **show** that the equation $e^x = 6-8x$ has a root in the interval $(0,1)$.

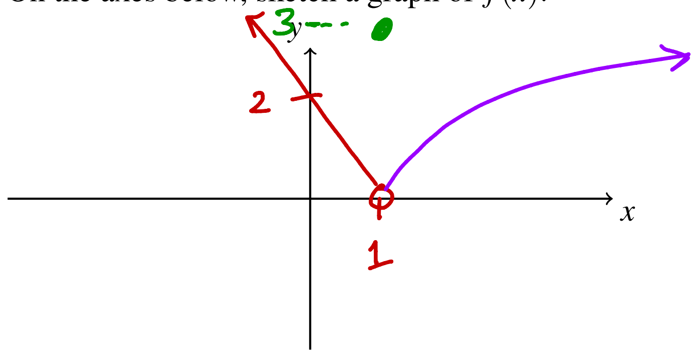
Let $f(x) = e^x + 8x - 6$. Observe that $f(x)$ is continuous.

Now $f(0) = e^0 + 8\cdot0 - 6 = -5 < 0$ and $f(1) = e^1 + 8\cdot1 - 6 = e+2 > 0$.

Since $f(0)<0$ and $f(1)>0$, $f(x)$ must equal zero somewhere in $(0,1)$.

3. [8 points] Consider the function $f(x) = \begin{cases} 2-2x & x<1 \\ 3 & x=1 \\ \sqrt{x-1} & x>1. \end{cases}$

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate the limit below or explain why the limit fails to exist.

$\lim_{x\to 1} f(x) = 0$

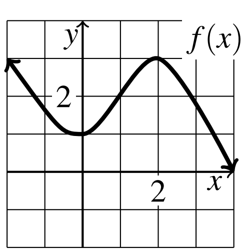
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \sqrt{x-1} = 0$ and $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2-2x = 0$

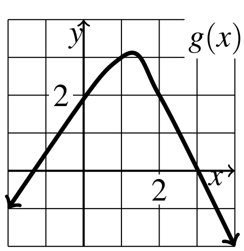
Both one-sided limits are equal.

c. Is f continuous at $x=1$? Explain using the definition of continuity.

No. $\lim_{x\to 1} f(x) = 0 \neq 3 = f(0)$.

4. [4 points] The graphs of $f(x)$ and $g(x)$ are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x\to 2} \frac{5f(x)}{2+g(x)} = \frac{5\cdot 3}{2+2} = \frac{15}{4}$

b. $\lim_{x\to 2} 4x + f(x) = 4\cdot 2 + 3 = 11$