

Name: _____

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Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit.

1. [12 points] Differentiate the functions. Write your answer using appropriate derivative notation, but you need not simplify your answers.

a. $g(u) = u^{1/3} - u^{4/3}$

$$g'(u) = \frac{1}{3} u^{-2/3} - \frac{4}{3} u^{-1/3}$$

b. $f(x) = \frac{2}{x^3} = 2x^{-3}$

$$f'(x) = -6x^{-4}$$

c. $h(x) = x^{e-1} + \frac{1}{e^2}$

$$h'(x) = (e-1)x^{e-2}$$

d. $s(t) = (4-t)e^t$

$$\begin{aligned} s'(t) &= (-1)e^t + (4-t)e^t \\ &= e^t(3-t) \end{aligned}$$

e. $F(t) = \frac{At}{B+Ct^2}$

$$\begin{aligned} F'(t) &= \frac{A(B+Ct^2) - At(2Ct)}{(B+Ct^2)^2} \\ &= \frac{AB - Act^2}{(B+Ct^2)^2} \end{aligned}$$

2. [4 points] Suppose that $f(2) = 5$, $g(2) = 1$, $f'(2) = -3$, and $g'(2) = 4$. Find the following values.

$$\begin{aligned} \text{a. } (fg)'(2) &= f'(2)g(2) + f(2)g'(2) \\ &= (-3)(1) + (5)(4) = \boxed{17} \end{aligned}$$

$$\begin{aligned} \text{b. } \left(\frac{f}{g}\right)'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\ &= \frac{(-3)(1) - (5)(4)}{1^2} = \boxed{-23} \end{aligned}$$

3. [6 points] The equation of motion of a particle is $s = t^4 - 2t^3 - 3$, where s is in meters and t is in seconds. **Include the units for each answer.**

a. What is the acceleration as a function of t ?

$$s'(t) = v(t) = s'(t) = 4t^3 - 6t^2$$

$$a(t) = s''(t) = \boxed{12t^2 - 12t \frac{\text{m}}{\text{s}^2}}$$

b. Find the velocity at the time $t > 0$ when the acceleration is 0.

$$a(t) = 0 \Leftrightarrow 12t^2 - 12t = 0 \Leftrightarrow 12t(t-1) = 0$$

$$\Leftrightarrow t = 0, t = 1$$

$$v(1) = 4 \cdot 1 - 6 \cdot 1 = \boxed{-2 \frac{\text{m}}{\text{s}}}$$

4. [3 points] For what value of x does the graph of $f(x) = 2e^x - 5x$ have a horizontal tangent?

$$f'(x) = 2e^x - 5 = 0 \Leftrightarrow \boxed{x = \ln\left(\frac{5}{2}\right)}$$