CORRECTED!

Math 251: Quiz 4 SOLUTIONS **18 February, 2020**

Name: ______________________ ________ / 25

Circle one: **Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)**

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit.

1. **[12 points]** Differentiate the functions. Write your answer using appropriate derivative notation, but you need not simplify your answers.

   a. $f(x) = \dfrac{3}{x^2}$

   $$f'(x) = 3(-2)x^{-3} = -6x^{-3}$$

   b. $g(u) = u^{1/3} - u^{5/3}$

   $$g'(u) = \frac{1}{3}u^{-2/3} - \frac{5}{3}u^{2/3}$$

   c. $h(x) = x^{e-1} + \frac{1}{e^3}$

   $$h'(x) = (e-1)x^{e-2}$$

   d. $F(t) = \dfrac{at}{b+ct^2}$

   $$F'(t) = \frac{a(b+ct^2) - at(2ct)}{(b+ct^2)^2} = \frac{ab - act^2}{(b+ct^2)^2}$$

   e. $s(t) = e^t(5-t)$

   $$s'(t) = e^t(5-t) + e^t(-1)$$
   $$= e^t(4-t)$$

2. [4 points] Suppose that $f(3) = 5$, $g(3) = -1$, $f'(3) = -4$, and $g'(3) = 3$. Find the following values.

a. $(fg)'(3)$ $= f'(3)\,g(3) + f(3)\,g'(3) = (-4)(-1) + (5)(3)$

$= 19$

b. $\left(\frac{f}{g}\right)'(3)$ $= \dfrac{f'(3)\,g(3) - f(3)\,g'(3)}{g(3)^2} = \dfrac{(-4)(-1)-(5)(3)}{(-1)^2}$

$= -11$

3. [6 points] The equation of motion of a particle is $s = t^4 - 2t^3 - 4$, where s is in meters and t is in seconds. Include the units for each answer.

a. What is the acceleration as a function of t?

$v(t) = s'(t) = 4t^3 - 6t^2$

$a(t) = 12t^2 - 12t \ \frac{m}{s^2}$

b. Find the velocity at the time $t > 0$ when the acceleration is 0.

$a(t) = 0 \iff 12t^2 - 12t = 0 \iff 12t(t-1) = 0$

$\iff t = 0, \underline{\underline{1}}$

$v(1) = 4 \cdot 1^3 - 6 \cdot 1^2 = -2 \ \frac{m}{s}$

4. [3 points] For what value of x does the graph of $f(x) = 5e^x - 2x$ have a horizontal tangent?

$f'(x) = 5e^x - 2 \quad \therefore \quad 0 = 5e^x - 2 \iff x = \ln\left(\frac{2}{5}\right)$