

Name: _____

/ 25

Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [15 points] Differentiate the following. Use proper notation to indicate your answer.

a. $f(x) = (2x-5)^2(x^2+4)^3$

$$f'(x) = 2(2x-5)(2)(x^2+4)^3 + (2x-5)^2(3)(x^2+4)^2(2x)$$

$$= 2(2x-5)(x^2+4)^2 [2(x^2+4) + 3x(2x-5)]$$

$$f'(x) = 2(2x-5)(x^2+4)^2 [8x^2 - 15x + 8]$$

b. $g(x) = 10^{2 \tan x}$

$$g'(x) = 10^{2 \tan x} \cdot \ln(10) \cdot 2 \sec^2 x$$

c. $f(x) = x^3 e^{-1/x}$

$$f'(x) = 3x^2 e^{-1/x} + x^3 e^{-1/x} \left(\frac{1}{x^2}\right)$$

$$f'(x) = 3x^2 e^{-1/x} + x e^{-1/x}$$

d. $h(x) = \frac{\cos(x)}{1-x^2}$

$$h'(x) = \frac{(1-x^2)(-\sin x) - \cos x(-2x)}{(1-x^2)^2}$$

$$h'(x) = \frac{(x^2-1)(\sin x) + 2x \cos x}{(1-x^2)^2}$$

e. $f(t) = \sqrt{3t - \sin^2 t}$

$$f'(t) = \frac{1}{2} (3t - \sin^2 t)^{-1/2} (3 - 2 \sin t \cos t)$$

$$f'(t) = \frac{3 - 2 \sin t \cos t}{2 \sqrt{3t - \sin^2 t}}$$

2. [6 points] The amount of water in a tank t minutes after it has started to drain is given by

$W = 10(t - 15)^2$ gal. Be sure to include proper units in your answers.

- a. How many gallons of water are in the tank at time $t = 0$?

$$W(0) = 10(0 - 15)^2 = 10(-15)^2 = 10(225) = \boxed{2250 \text{ gal}}$$

- b. At what rate is the water running out at the end of 5 minutes?

$$W'(t) = 20(t - 15)$$

$$W'(5) = 20(5 - 15) = 20(-10) = \boxed{-200 \text{ gal/min}}$$

- c. What is the average rate at which the water flows out during the first 5 minutes?

$$\frac{W(5) - W(0)}{5 - 0} = \frac{10(5 - 15)^2 - 2250}{5} = \frac{-1250}{5}$$

$$= \boxed{-250 \text{ gal/min}}$$

3. [4 points] Find an equation of the tangent line to the curve $y = \frac{8}{4 + \tan x}$ at the point where $x = \pi/4$.

$$y' = -8(4 + \tan x)^{-2} \cdot \sec^2 x$$

$$y'(\pi/4) = -8(4 + \tan \pi/4)^{-2} \cdot \sec^2(\pi/4)$$

$$= -8(5)^{-2} \cdot \left(\frac{2}{\sqrt{2}}\right)^2$$

$$= -\frac{8}{25} \cdot \frac{4}{2} = -\frac{16}{25}$$

$$y(\pi/4) = \frac{8}{4+1} = \frac{8}{5}$$

$$\boxed{y - \frac{8}{5} = -\frac{16}{25}(x - \pi/4)}$$