Math 251: Quiz 5 Solutions
Name: ______

February 25, 2020

______ / 25

Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. **[15 points]** Differentiate the following. Use proper notation to indicate your answer.

a. $h(x) = \dfrac{\cos(x)}{1-x^2}$

$$h'(x) = \frac{(1-x^2)(-\sin x) - (\cos x)(-2x)}{(1-x^2)^2}$$

$$\boxed{h'(x) = \frac{(x^2-1)(\sin x) + 2x\cos x}{(1-x^2)^2}}$$

b. $f(x) = (2x-5)^3(x^2+4)^2$

$$f'(x) = 3(2x-5)^2(2)(x^2+4)^2 + (2x-5)^3(2)(x^2+4)(2x)$$

$$= 2(2x-5)^2(x^2+4)\left[3(x^2+4) + 2x(2x-5)\right]$$

$$\boxed{f'(x) = 2(2x-5)^2(x^2+4)\left[7x^2-10x+12\right]}$$

c. $g(x) = 10^{2\tan x}$

$$\boxed{g'(x) = \left(10^{2\tan x}\right)(\ln 10)(2\sec^2 x)}$$

d. $f(t) = \sqrt{2t - \sin^3 t}$

$$f'(t) = \frac{1}{2}(2t - \sin^3 t)^{-1/2}(2 - 3\sin^2 t\cos t)$$

$$\boxed{f'(t) = \frac{2 - 3\sin^2 t\cos t}{2\sqrt{2t-\sin^3 t}}}$$

e. $f(x) = x^3 e^{-1/x}$

$$f'(x) = 3x^2 e^{-1/x} + x^3 e^{-1/x}\left(\frac{1}{x^2}\right)$$

$$\boxed{f'(x) = 3x^2 e^{-1/x} + x e^{-1/x}}$$

2. [6 points] The amount of water in a tank t minutes after it has started to drain is given by $W = 10(t-10)^2$ gal. Be sure to include proper units in your answers.

a. How many gallons of water are in the tank at time $t = 0$?

$$W(0) = 10(0-10)^2 = 10(100) = \boxed{1000 \text{ gallons}}$$

b. At what rate is the water running out at the end of 5 minutes?

$$W'(t) = 20(t-10)$$

$$W'(5) = 20(5-10) = 20(-5) = \boxed{-100 \text{ gal/min}}$$

c. What is the average rate at which the water flows out during the first 5 minutes?

$$\frac{W(5)-W(0)}{5-0} = \frac{10(5-10)^2 - 1000}{5-0} = -\frac{750}{5} = \boxed{-150 \text{ gal/min}}$$

3. [4 points] Find an equation of the tangent line to the curve $y = \dfrac{8}{4+\tan x}$ at the point where $x = 0$.

$$y' = -8(4+\tan x)^{-2} \cdot \sec^2 x$$

$$y'(0) = -8(4+\tan 0)^{-2} \cdot \sec^2(0)$$

$$= -8(4)^{-2} \cdot 1$$

$$= -\frac{8}{16} = -\frac{1}{2}$$

$$y(0) = \frac{8}{4+0} = 2$$

$$\boxed{y - 2 = -\tfrac{1}{2}(x)}$$