Math 251: Quiz 7 — SOLUTIONS — 31 March, 2020

Name: _____ / 10

Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

10 points possible. **No aids (internet, other students, book, calculator, etc.) are permitted.** You do not need to simplify final answers, but **answers without supporting work will lose points for completeness and effort.**

1. **[3 points]** Find the absolute maximum and absolute minimum values of f on the given interval. State the answer as points; give both the x- and y-coordinates of the extrema.

$f(x) = 1 + 12x - x^3, \qquad [0,3]$

$f'(x) = 12 - 3x^2 \stackrel{?}{=} 0$

$x^2 = 4$

$x = \pm 2$

$x = +2$ is in $[0,3]$

x	$f(x)$	
0	1	abs. min
2	17	
3	10	abs. max

2. **[2 points]** Consider the function $g(t) = te^{-t^2}$.

a. Find all of the critical numbers.

$g'(t) = 1 \cdot e^{-t^2} + t \cdot e^{-t^2}(-2t)$

$= (1 - 2t^2)e^{-t^2}$

$e^{\text{anything}} \neq 0$ ← remember

$1 - 2t^2 = 0$

$t = \pm\sqrt{\frac{1}{2}}$

b. Find the x-coordinate all of the inflection points.

$g''(t) = -4t \cdot e^{-t^2} + (1-2t^2)e^{-t^2}(-2t)$

$= (-4t - 2t + 4t^3)e^{-t^2}$

$= (4t^3 - 6t)e^{-t^2}$

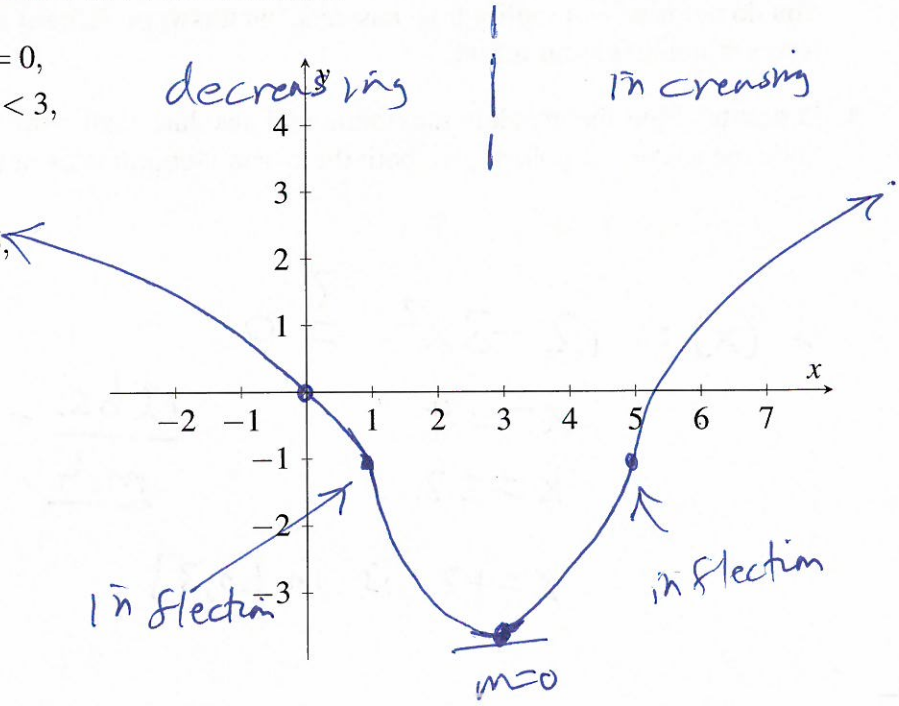
$4t^3 - 6t = 0$

$t = 0$ $\qquad 2t^2 - 3 = 0$

$t = \pm\sqrt{\frac{3}{2}}$

3. [3 points] Sketch a graph that satisfies all of the conditions:

domain $f = (-\infty, \infty)$, $f(0) = 0$,
$f'(3) = 0$, $f'(x) < 0$ when $x < 3$,
$f'(x) > 0$ when $x > 3$,
$f''(1) = 0$, $f''(5) = 0$,
$f''(x) < 0$ when $x < 1$ or $x > 5$,
$f''(x) > 0$ for $1 < x < 5$



4. [2 points] Consider the function $f(x) = x \ln x$.

a. What is the domain of f?

$x > 0$ [or: $(0, \infty)$]

b. Find the intervals of increase and decrease

$$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1 \stackrel{?}{=} 0$$

$$\ln x = -1$$
$$x = e^{-1} = \frac{1}{e}$$

x	$f'(x)$
$1/10$	$-$
$1/e$	0
1	$+$

decreasing on $(0, \frac{1}{e})$

increasing on $(\frac{1}{e}, \infty)$