Math 251: Quiz 8 **OPTIONAL** **DUE: 6 April, 2020 by 5PM**

Name: SOLUTIONS ____________ / 10

Circle one: **Faudree ~~(F01)~~ | ~~Bueler (F02)~~ | VanSpronsen (UX1)**

This OPTIONAL Quiz is worth 10 points. The purpose is to get some additional practice and additional feedback prior to Midterm 2. **No aids (internet, other students, book, calculator, etc.) are permitted.** You do not need to simplify final answers, but **answers without supporting work will lose points for completeness and effort.**

1. [4 points] Answer the questions below for the function $f(x) = e^{2x} - 4e^x + 1$.

a. Evaluate $\lim_{x\to-\infty} f(x)$ and explain what this implies about the graph of $f(x)$.

$$\lim_{x\to-\infty} e^{2x} - 4e^x + 1 = 0 - 0 + 1 = 1 \quad \therefore$$

$y = 1$ is a horizontal asymptote

b. Determine the intervals of increase or decrease and identify the x-values of any local extrema. State whether they are maxima or minima.)

$$f'(x) = 2e^{2x} - 4e^x = 0 \iff 2e^x(e^x - 2) = 0$$
$$\iff e^x = 2 \iff x = \ln 2$$

increasing: $(\ln 2, \infty)$

decreasing: $(-\infty, \ln 2)$

loc. min. @ $x = \ln 2$

x	$f'(x)$
0	$-$
$\ln 2$	0
1	$+$

c. Determine the concavity of the graph of f and find the x-values of any points of inflection.

$$f''(x) = 4e^{2x} - 4e^x = 0 \iff 4e^x(e^x - 1) = 0$$
$$\iff e^x = 1 \iff x = 0$$

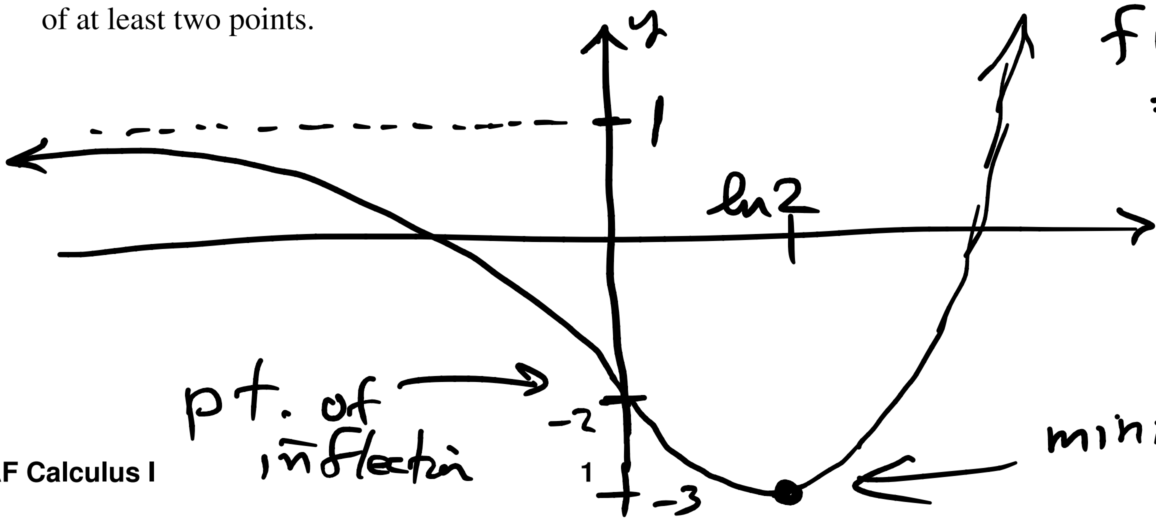
Concave up: $(0, \infty)$

Concave down: $(-\infty, 0)$

inflection pt: $x = 0$

x	$f''(x)$
	$-$
0	0
	$+$

d. Use the information about to sketch the graph of f. Your graph should give the coordinates of at least two points.



$f(\ln 2) = 2^2 - 4\cdot 2 + 1 = -3$

2. [2 points] Find the limit. Use l'Hospital's Rule where appropriate.

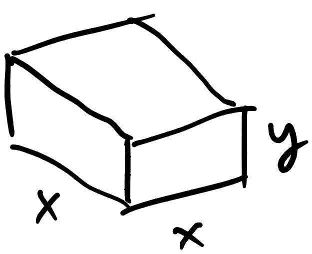
a. $\lim_{x\to 1} \frac{x^p - 1}{2x - 2}$, where p is a fixed constant.

$$\overset{\frac{0}{0}}{\underset{L'H.}{=}} \lim_{x\to 1} \frac{p x^{p-1}}{2} = \frac{p}{2}$$

b. $\lim_{x\to\infty} x \ln\left(1 + \frac{5}{x}\right)$ $=$ "$\infty \cdot 0$"

$$\lim_{x\to\infty} x \ln\left(1+\frac{5}{x}\right) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{5}{x}\right)}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{1}{1+5/x}\cdot\frac{-5}{x^2}}{\frac{-1}{x^2}} = \lim_{x\to\infty} \frac{5}{1+5/x} = \frac{5}{1} = 5$$

3. [4 points] A rectangular box with square base must have volume 20 cubic meters. Material for the base and sides costs \$ 2 per square meter. Material for the top costs \$6 per ~~cubic~~ square meter. Find the dimensions of the least expensive box.



$$20 = x^2 y$$

$$C = 2(\underbrace{x^2}_{\text{bottom}} + \underbrace{4xy}_{\text{4 sides}}) + \underbrace{6x^2}_{\text{top}}$$

$$= 8x^2 + 8xy = 8x^2 + 8x\frac{20}{x^2}$$

$$\therefore\ C(x) = 8x^2 + \frac{160}{x}$$

$$C'(x) = 16x - \frac{160}{x^2} = 0$$

$$16x^3 = 160$$

$$x^3 = 10$$

$$x = \sqrt[3]{10}$$

$$y = \frac{20}{x^2} = \frac{20}{10^{2/3}}$$

$$y = 2\sqrt[3]{10}$$

x	$C(x)$
0	∞
$\sqrt[3]{10}$	min.
∞	∞