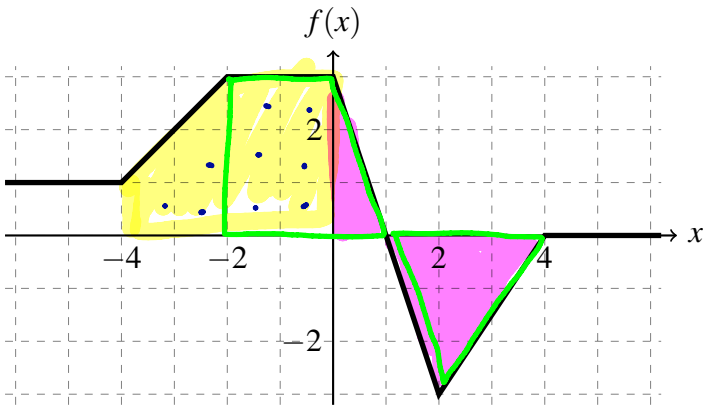
Name: Solutions _____ / 25

Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

25 points possible. **No aids (internet, other students, book, calculator, etc.) are permitted.** You do not need to simplify final answers, but **answers without supporting work will lose points for completeness and effort.**

1. [6 points] The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



a. $\int_{-4}^{0} f(x)\,dx =$ yellow area = 10

b. $\int_{0}^{4} f(x)\,dx =$ area above − area below $= \frac{3}{2} - \frac{3\cdot 3}{2} = -\frac{6}{2} = -3$

c. $\int_{4}^{-2} f(x)\,dx = -\int_{-2}^{4} f(x)\,dx = -\left(6 + (-3)\right) = -3$

2. [6 points] A particle is moving with the given acceleration $a(t)$ and other data. Find the position $s(t)$ of the particle.

$a(t) = \sin t + \cos t, \quad s(0) = 3, \quad v(0) = 4$

antiderivative: $v(t) = -\cos t + \sin t + C$

Substitute: $4 = v(0) = -\cos 0 + \sin 0 + C = -1 + C$. So $C = 5$

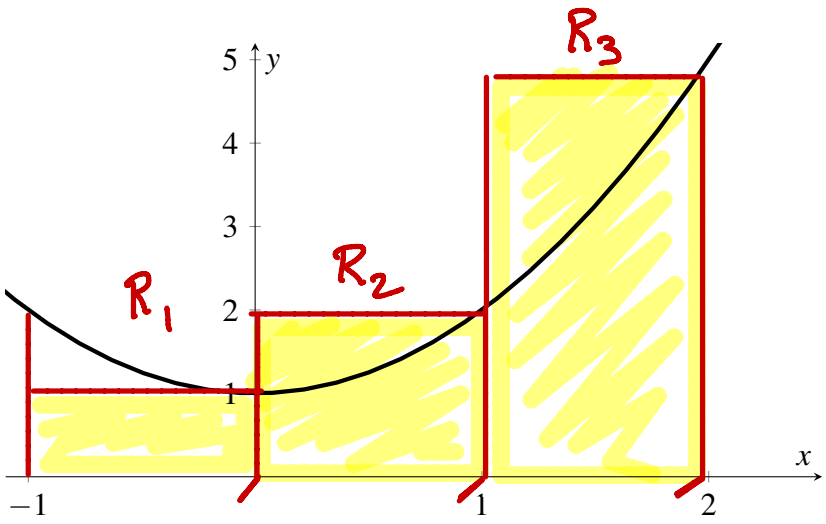
antiderivative: So $v(t) = -\cos t + \sin t + 5$

So $s(t) = -\sin t - \cos t + 5t + C$.

Substitute: $3 = s(0) = -\sin 0 - \cos 0 + 5\cdot 0 + C = -1 + C$. So $C = 4$

Answer: $s(t) = -\sin t - \cos t + 5t + 4$

3. [8 points] Consider the graph of $f(x) = 1 + x^2$ below.



a. In the figure above, sketch three rectangles corresponding to the $n = 3$ Riemann sum on the interval $-1 \le x \le 2$. Use right endpoints.

b. Compute the numerical value of the Riemann sum illustrated in part **a**. Express your answer as an integer.

$1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2)$

$= 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8$

c. Is your numerical value in part **b** an overestimate or an underestimate of $\int_{-1}^{2} 1 + x^2\,dx$?

overestimate (See how much yellow is above curve.)

4. [5 points] Evaluate the integral by interpreting it in terms of areas:

$\int_{-2}^{3} |\frac{1}{2}x|\,dx = \frac{1}{2} \cdot 1 \cdot 2 + \frac{1}{2} \cdot \frac{3}{2} \cdot 3$

$= 1 + \frac{9}{4} = \frac{13}{4}$