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There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] For each function below, find its derivative. You do not need to simplify your answer.

a. $y = 2x^{3/4} + \pi^2$

$y' = (2 \cdot x^{3/4})' + (\pi^2)' = 2 \cdot \frac{3}{4} \cdot x^{-1/4} + 0 = \boxed{\frac{3}{2} x^{-1/4}}$

b. $f(x) = \frac{1}{2x^2} - x^e$

$f'(x) = \left(\frac{1}{2x^2}\right)' - (x^e)' = -\frac{2}{2x^3} - e x^{e-1} = \boxed{-\frac{1}{x^3} - e x^{e-1}}$

c. $g(t) = e^t(\sqrt{t} + 2t)$

$g'(t) = (e^t)'(\sqrt{t} + 2t) + e^t(\sqrt{t} + 2t)' =$

$= \boxed{e^t(\sqrt{t} + 2t) + e^t\left(\frac{1}{2\sqrt{t}} + 2\right)}$

d. $y = \frac{2 - 3x}{e^x + x}$

$y' = \frac{(2-3x)'(e^x+x) - (e^x+x)'(2-3x)}{(e^x+x)^2} = \frac{-3(e^x+x) - (e^x+1)(2-3x)}{(e^x+x)^2} =$

$= \frac{-3e^x - 3x - 2e^x + 3xe^x - 2 + 3x}{(e^x+x)^2} = \boxed{\frac{-5e^x + 3xe^x - 2}{(e^x+x)^2}}$

e. $h(x) = \frac{2}{1+x^2}$

$h'(x) = \frac{0 \cdot (1+x^2) - 2 \cdot 2x}{(1+x^2)^2} = \boxed{\frac{-4x}{(1+x^2)^2}}$

2. [5 points] The radius of a balloon being inflated is described by the function

$$r(t) = 2 + 5t^{\frac{1}{3}}$$

where r is measured in centimeters and t in seconds.

a. What is the radius of the balloon at time $t = 1$? Include **units** in your answer.

$r(1) = 2 + 5 \cdot 1 = \boxed{7 \text{ (cm)}}$

b. What is the rate of change of the radius at time $t = 1$? Include **units** in your answer.

$r'(t) = (2 + 5t^{1/3})' = 5 \cdot \frac{1}{3} t^{-2/3}$

At $t=1$: $r'(1) = \boxed{\frac{5}{3} \text{ (cm/sec)}}$

3. [5 points] Find the equation of the tangent line to the curve $y = \frac{3}{x} + 6$ at $x = -2$.

Tangent line equation: $y(x) = y'(a)(x-a) + y(a)$

$a = -2$

$y(a) = y(-2) = -\frac{3}{2} + 6 = \frac{-3+12}{2} = \frac{9}{2}$

$y'(x) = \left(\frac{3}{x} + 6\right)' = -\frac{3}{x^2}$

$y'(-2) = -\frac{3}{(-2)^2} = -\frac{3}{4}$

Therefore,

$y(x) = -\frac{3}{4}(x+2) + \frac{9}{2} = -\frac{3}{4}x + 3$

$\boxed{y = -\frac{3}{4}x + 3}$