Math 251: Quiz 5 February 23, 2021

Name: Solutions ______ / 20

20 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [8 points] For each function below, find its derivative. **No credit** will be given unless sufficient work is given to justify your answer. You do not need to simplify your answer.

a. $f(x) = x\sin(x) + 3\tan(x)$

$$f'(x) = \sin(x) + x\cdot\cos(x) + 3\cdot\sec^2(x)$$

b. $g(t) = (t^3-2)^2\sec(t)$

$$g'(t) = 2(t^3-2)\cdot 3t^2\sec(t) + (t^3-2)^2\cdot\sec(t)\cdot\tan(t) =$$

$$= 6t^2(t^3-2)\sec(t) + (t^3-2)^2\sec(t)\tan(t)$$

c. $f(x) = x^3e^{-1/x}$

$$f'(x) = 3x^2\cdot e^{-\frac{1}{x}} + x^3\cdot e^{-\frac{1}{x}}\cdot\left(\frac{1}{x^2}\right) =$$

$$= 3x^2\cdot e^{-\frac{1}{x}} + x\cdot e^{-\frac{1}{x}}$$

d. $s(t) = \dfrac{\cos(3t^2)}{1-t}$

$$s'(t) = \frac{-\sin(3t^2)\cdot 6t\cdot(1-t) - \cos(3t^2)(-1)}{(1-t)^2}$$

2. [8 points] A circular blot of ink is growing. Its radius r in centimeters at time $t \geq 0$ seconds is

$$r(t) = 5 - \frac{9}{2}(1+t)^{-2}$$

a. What is the radius of the blot at time $t = 2$ second? Your answer should include **units**.

$$r(2) = 5 - \frac{9}{2}(1+2)^{-2} = 5 - \frac{\cancel{9}}{2} \cdot \frac{1}{\cancel{9}} = \boxed{4.5}\ (cm)$$

b. What is the average rate of change of the radius of the blot from time $t = 0$ to time $t = 2$ seconds? Be sure to include **units** in your answer.

$$\frac{\Delta r}{\Delta t} = \frac{r(t_2) - r(t_1)}{t_2 - t_1} = \frac{r(2) - r(0)}{2} = \frac{4.5 - 0.5}{2} = \boxed{2}\ (cm/sec)$$

$$r(2) = 4.5 \quad ; \quad r(0) = 5 - \frac{9}{2} = 0.5$$

c. What is the instantaneous rate of change of the radius of the blot at time $t = 1$ second? Again, include **units** in your answer.

$$r'(t) = -\frac{9}{2} \cdot (-2)(1+t)^{-3} = 9(1+t)^{-3}$$

$$r'(1) = 9(2)^{-3} = \boxed{\frac{9}{8}}\ (cm/sec)$$

3. [4 points] The position of an object is $s(t) = \sqrt{2t^2 - 3t + 8}$ meters at time $t \geq 0$ seconds. At what time, if any, is the instantaneous velocity of the particle equal to 0?

$$s'(t) = \frac{1}{2\sqrt{2t^2 - 3t + 8}} \cdot (4t - 3) = 0$$

$4t - 3 = 0$, and $2t^2 - 3t + 8 > 0$

$4t = 3$

$2(t^2 - \frac{3}{2}t + 4) =$

$= (t - \frac{3}{4})^2 + 4 - \frac{9}{16} =$

$= (t - \frac{3}{4})^2 + \frac{55}{16} > 0$ for all $t \geq 0$

$$\boxed{t = \frac{3}{4}\ (sec)}$$