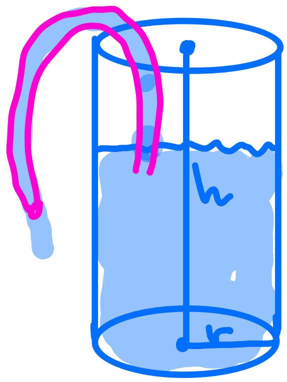
Math 251: Quiz 6 **March 16, 2021**

Name: Solutions ________ / **20**

20 points possible. A scientific or graphing calculator is permitted (for problem #3 in particular), but no other aids are allowed. Show all work and use proper notation for full credit.

1. [6 points] The fluid in a cylindrical tank with radius 5 m is draining at a rate of 2 m^3/min. How fast is the height of the water changing?



$r = 5m$

$\frac{dV}{dt} = -2\, m^3/min$

$\frac{dh}{dt} - ?$

$V = \pi r^2 \cdot h$

$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$

$\Downarrow$

$\frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{1}{\pi r^2}$

$\frac{dh}{dt} = -2 \cdot \frac{1}{\pi 25} = \frac{-2}{25\pi}\ m/min$

Answer: the height of water is decreasing at rate $\frac{2}{25\pi}\ m/min$

2. [6 points]

a. Find the linearization of $f(x) = \sqrt{x}$ at $a = 16$.

$L(x) = f'(a)(x-a) + f(a)$

$f(a) = 4$

$f'(a) = \frac{1}{2\sqrt{a}}$

$f'(16) = \frac{1}{8}$

$L(x) = \frac{1}{8}(x-16) + 4 = \frac{1}{8}x + 2$

b. Use part **a.** to estimate $\sqrt{17}$.

$\sqrt{17} \approx L(17) = \frac{1}{8}(17-16) + 4 = \frac{1}{8} + 4 = \frac{33}{8} \approx 4.125$

3. [8 points] Suppose the temperature of a liquid substance (in $^\circ$ F) undergoing a chemical reaction t minutes after its start is modeled by the function

$$T(t) = \ln(t^2 - t + 1).$$

a. What is the initial temperature of the liquid?

$T(0) = \ln(1) = 0\ (^\circ F)$

b. Find the time t from $[0,10]$ where the temperature T is at its maximum and minimum values.

$$T'(t) = \frac{1}{t^2-t+1}\cdot(2t-1) = \frac{2t-1}{t^2-t+1} =$$

$$= \frac{2t-1}{(t-\frac{1}{2})^2 - \frac{1}{4}+1} = \frac{2t-1}{\underbrace{(t-\frac{1}{2})^2+\frac{3}{4}}_{>0}}$$

$T'(t)$ exists for all t in $[0,10]$

CP

$T'(t) = 0 \Rightarrow \frac{2t-1}{t^2-t+1} = 0 \Rightarrow 2t = 1 \Rightarrow \boxed{t = \frac{1}{2}}$

$T(\frac{1}{2}) = \ln(\frac{1}{4} - \frac{1}{2} + 1) = \ln(0.75) \approx \boxed{-0.29\ (^\circ F)}$

$T(0) = \boxed{0\ (^\circ F)}$

$T(10) = \ln(100-10+1) = \ln(91) \approx \boxed{4.51\ (^\circ F)}$

Answer: at $t = \frac{1}{2}$ (min) $T_{min} = -0.29\ (^\circ F)$

at $t = 10$ (min) $T_{max} = 4.51\ (^\circ F)$