Name: Solutions ____ / 20

20 points possible. No aids are allowed. Show all work and use proper notation for full credit.

1. [6 points] Compute, with justification, the following limits:

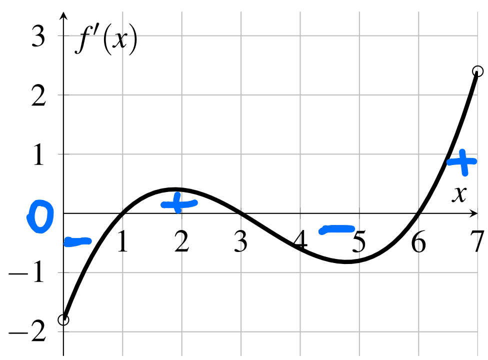
a. $\lim\limits_{t\to 0} \frac{\sin(t^2)}{t^2}$ = "$\frac{0}{0}$"

$$\lim_{t\to 0} \frac{\sin(t^2)}{t^2} \overset{L'H}{=} \lim_{t\to 0} \frac{\cos(t^2)\cdot \cancel{2t}}{\cancel{2t}} = \boxed{1}$$

b. $\lim\limits_{x\to\infty} e^{-x}\ln(x)$ = "$0\cdot\infty$"

$$\lim_{x\to\infty} e^{-x}\cdot\ln(x) = \lim_{x\to\infty} \frac{\ln(x)}{e^x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \lim_{x\to\infty} \frac{1}{xe^x} = \boxed{0}$$

2. [4 points] The graph of the **derivative** f' of a function f is shown.



a. On what intervals is f increasing or decreasing? Use interval notation.

$f(x)$ is increasing where $f'(x)>0$: $(1,2)\cup(6,7)$

$f(x)$ is decreasing where $f'(x)<0$: $(0,1)\cup(3,6)$

b. At what values of x in the open interval $(0,7)$ does f have a local maximum or minimum?

$f(x)$ has a loc. max at $x=3$

$f(x)$ has a loc. min at $x=1$ and $x=6$

3. [10 points] Consider the function $f(x) = xe^x$

a. Show that $f'(-1) = 0$.

$$f'(x) = xe^x + e^x = e^x(1+x)$$
$$f'(-1) = e^{-1}(1-1) = 0$$

b. Use the first derivative test to determine if a local minimum, a local maximum, or neither occurs at $x = -1$.

$-$ $+$ $f'(x)$

$\searrow$ -1 $\nearrow$ $f(x)$

1. $f'(-1) = 0$
2. $f'(x)$ is changing its sign from $\oplus$ to $\ominus$ near CP $x=-1$. Therefore, $f(x)$ has a loc. min at $x=-1$ and it is $f(-1) = -\frac{1}{e}$.

c. Is $f(x)$ concave up, concave down, or neither at $x = -1$?

$$f''(x) = (e^x(x+1))' = e^x(x+1) + e^x = e^x(x+2)$$
$$f''(-1) = e^{-1}(-1+2) = \frac{1}{e} > 0.$$

Therefore, $f(x)$ is concave up at $x=-1$

d. What does the previous answer tell you about the critical number $x = -1$?

Based on part (c) we have that at $x=-1$ the function $f(x)$ attains its loc. min value.

e. Determine any points of inflection of $f(x)$.

$$f''(x) = 0 \Rightarrow \underbrace{e^x}_{\neq 0}(x+2) = 0 \Rightarrow \boxed{x=-2}$$

Therefore,

$\boxed{x=-2}$ $\boxed{f(-2) = \frac{-2}{e^2}}$

$-$ $+$ $f''(x)$

$\frown$ -2 $\smile$ $f(x)$

$\left(-2, -\frac{2}{e^2}\right)$ is the only one IP.