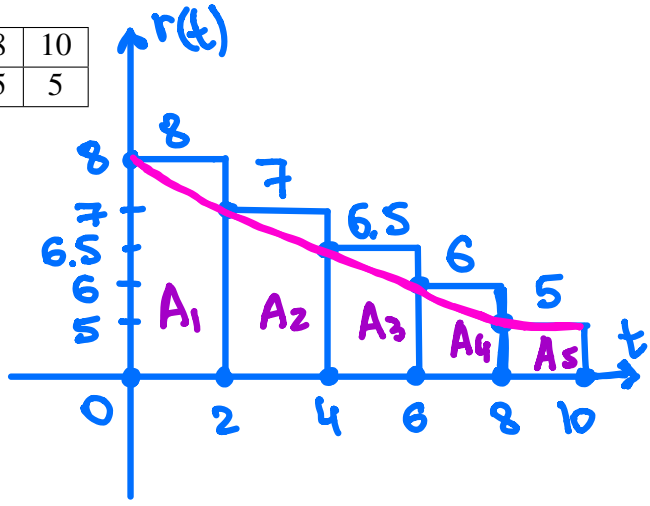
20 points possible. A graphing or scientific calculator is allowed. No aids are permitted. Show all work and use proper notation for full credit.

1. **[4 points]** Water is filling a tank at a rate of $r(t)$ liters per second over a ten second interval. The rate at 2 second time intervals are shown in the table. By using **left endpoints** for each two second time interval, estimate the amount of water that filled the tank in the 10 second interval. Include units in your answer.

t (sec)	0	2	4	6	8	10
r(t) (ℓ/sec)	8	7	6.5	6	5	5



Let A be the amount of water that filled the tank in the 10 sec interval. Then

$$A = \sum_{i=1}^{5} A_i = A_1 + A_2 + A_3 + A_4 + A_5$$
$$= 2(8+7+6.5+6+5) =$$
$$= 65 \ (\ell)$$

2. **[6 points]** In each case below, find a function f that satisfies the given criteria.

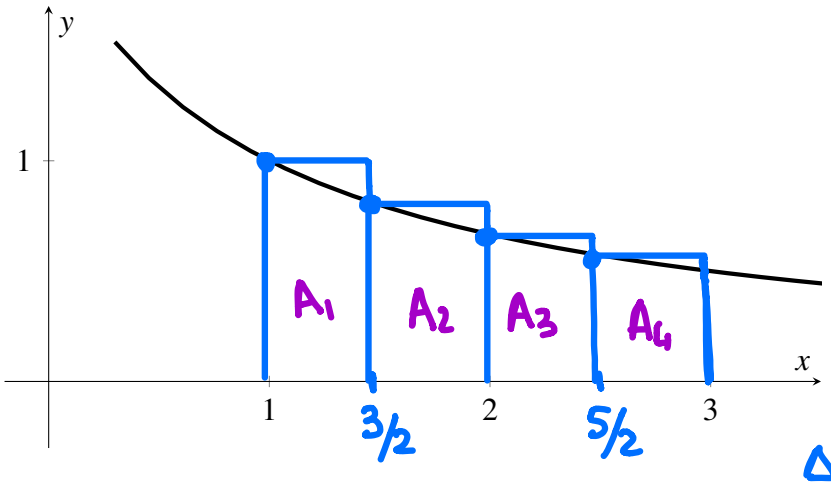
   a. $f'(t) = \sec(t)\tan(t) - 3e^t$

$$f(t) = \sec(t) - 3e^t + C$$

   b. $f'(t) = 1 - 2\sqrt{t}$

$$f(t) = t - \frac{2t^{3/2}}{3/2} + C = t - \frac{4}{3}t^{3/2} + C$$

3. [6 points] Consider the graph of $f(x) = \dfrac{2}{1+x}$ below.



width

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

a. In the figure above, sketch four rectangles corresponding to the $n = 4$ Riemann sum on the interval $1 \le x \le 3$. Use left endpoints.

b. Compute the numerical value of the Riemann sum illustrated in part **a**. Express your answer as either a single fraction or as a decimal correct to 5 significant digits. You may use a calculator, but you must show work justifying your computation.

Let A_i be the area of i-th rectangle. Then

$$\sum_{i=1}^{4} A_i = \sum_{i=1}^{4} f(x_i)\Delta x = f(1)\Delta x + f\left(\tfrac{3}{2}\right)\Delta x + f(2)\Delta x +$$

$$+ f\left(\tfrac{5}{2}\right)\Delta x = \frac{1}{2}\left(\frac{2}{1+1} + \frac{2}{1+3/2} + \frac{2}{1+2} + \frac{2}{1+5/2}\right) \approx 1.5$$

4. [4 points] A particle is moving with velocty $v(t) = 2\cos t - 3\sin t$ inches/second. At $t = 0$ the particle has position $s(0) = 5$ inches. Find the position $s(t)$ of the particle. Include units in your answer.

$$v(t) = s'(t) = 2\cos(t) - 3\sin(t)$$

$$s(t) = 2\sin(t) + 3\cos(t) + C$$

$$s(0) = 0 + 3 + C = 5 \Rightarrow C = 2$$

Therefore,

$$s(t) = 2\sin(t) + 3\cos(t) + 2 \text{ (inch.)}$$