Math 251: Quiz 9 **April 13, 2021**

Name: Solutions ______ / 25

25 points possible. A graphing or scientific calculator is allowed. No aids are permitted. Show all work and use proper notation for full credit.

1. **[9 points]** Compute the following definite integrals.

a. $\int_{-2}^{2} (4-x^2)\, dx$ $= \left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) =$

$= 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$

b. $\int_{0}^{\pi/2} \sin(t)\, dt$ $= -\cos(t)\Big|_{0}^{\frac{\pi}{2}} = -\cos\left(\frac{\pi}{2}\right) + \cos(0) =$

$= \boxed{1}$

c. $\int_{1}^{6} \frac{2+x^2}{\sqrt{x}}\, dx$ $= \int_1^6 \left(2x^{-1/2} + x^{3/2}\right) dx = \left[2\frac{x^{1/2}}{1/2} + \frac{x^{5/2}}{5/2}\right]\Bigg|_1^6 =$

$= \boxed{\left(4\sqrt{6} + \frac{2}{5}\sqrt{6^5}\right) - \left(4 + \frac{2}{5}\right)}$

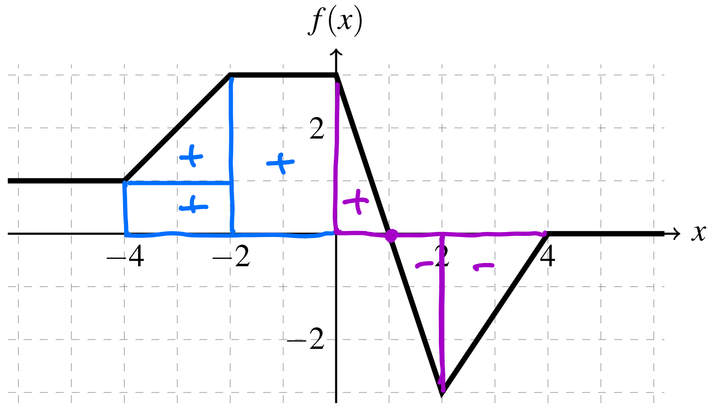
2. **[2 points]** Compute the derivative of the following function:

$$f(x) = \int_{0}^{2x} \sqrt{1+t^2}\, dt.$$

By the FTC part 1:

$f'(x) = \sqrt{1+(2x)^2} \cdot (2x)' = \boxed{\sqrt{1+4x^2} \cdot 2}$

3. [6 points] The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



a. $\int_{-4}^{0} f(x)\,dx = 6 + 2 + 2 = \boxed{10}$

b. $\int_{0}^{4} f(x)\,dx = -\frac{1}{2}\,3 \cdot 2 = \boxed{-3}$

c. $\int_{4}^{-2} f(x)\,dx = -\int_{-2}^{4} f(x)\,dx = 6 - 3 = \boxed{3}$

4. [8 points] Assuming $\int_1^5 f(x)\,dx = 3$, $\int_5^7 f(x)\,dx = -2$ and $\int_1^5 g(x)\,dx = 4$, compute the following.

a. $\int_1^5 2f(x)\,dx = 2\int_1^5 f(x)dx = 2 \cdot 3 = \boxed{6}$

b. $\int_5^5 f(x)\,dx = 0$

c. $\int_1^7 f(x)\,dx = \int_1^5 f(x)\,dx + \int_5^7 f(x)dx = 3 - 2 = \boxed{1}$

d. $\int_1^5 [f(x) - 2g(x)]\,dx = \int_1^5 f(x)\,dx - 2\int_1^5 g(x)dx = 3 - 2 \cdot 4 = \boxed{-5}$