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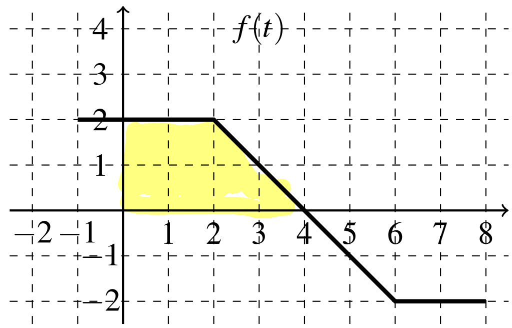
There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [4 points] Define $G(x) = \int_0^x f(t)\,dt$ where the graph of $f(t)$ is drawn below.



a. Determine $G(4)$.

$$G(4) = \boxed{\int_0^4 f(t)\,dt} = 6$$

b. Does $G(x)$ have a maximum on the interval $[0,8]$? Explain your answer.

Yes. G has a maximum at $x=4$ because $G' = f$ is positive on the left and negative on the right.

2. [6 points] Use the Fundamental Theorem of Calculus (Part 1) to find each derivative.

a. $\dfrac{d}{dx}\left(\int_1^x \ln(t)\,dt\right) = \ln(x)$

b. $\dfrac{d}{dx}\left(\int_{\cos(x)}^1 \sqrt{1-t^2}\,dt\right) = \left(\sqrt{1-\cos^2 x}\right)(-\sin(x))$

3. [8 points] Evaluate each definite integral using the Fundamental Theorem of Calculus Part 2.

a. $\int_1^{25} \frac{2}{\sqrt{x}}\,dx = \int_1^{25} 2x^{-1/2}\,dx = 2 \cdot 2x^{1/2}\Big]_1^{25}$

$= 4(\sqrt{25} - \sqrt{1}) = 4(5-1) = \boxed{16}$

b. $\int_0^{\pi/2} (5 - 3\sin(x))\,dx = 5x + 3\cos(x)\Big]_0^{\frac{\pi}{2}} = \left(\frac{5\pi}{2} + 3\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\right) - \left(0 + 3\underbrace{\cos(0)}_{1}\right)$

$= \boxed{\frac{5\pi}{2} - 3}$

4. [7 points] A ball is thrown upward from an initial height of $2\,m$ at an initial speed of $20\,m/s$. Acceleration resulting from gravity is $-9.8\,m/s^2$. (Just to be clear, we are assuming $a(t) = -9.8$ is the equation modeling the acceleration of the ball.)

a. Solve for $v(t)$, the velocity of the ball t seconds after it is thrown into the air.

$v(t) = \int a(t)\,dt = \int -9.8\,dt = -9.8t + C$

Use $v(0) = 20$.

So $v(0) = (-9.8)(0) + C = 20$

So, $C = 20$

$\boxed{v(t) = -9.8t + 20}$

b. Solve for $h(t)$, the height of the ball t seconds after it is thrown into the air.

$h(t) = \int v(t)\,dt = \int (-9.8t + 20)\,dt = -4.9t^2 + 20t + C$

$h(0) = 2$

So $h(0) = (-4.9)(0)^2 + 20(0) + C = 2$

So $C = 2$.

$\boxed{h(t) = -4.9t^2 + 20t + 2}$