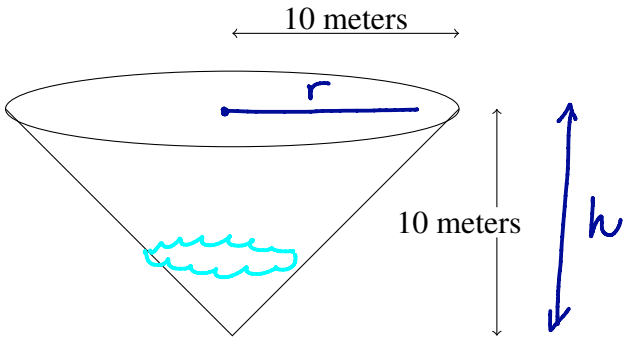
Name: Solutions _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[8 points]** A conical tank is filled with water at a rate of 20 cubic meters per second. The height and radius of the tank are both 10 meters. (See picture below.) How fast is the height of the water changing when the height of the water is 2 meters high? Include units with your answer. (Recall that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where V is volume, r is the radius of the top of the cone and h is the height of the cone.)



$\frac{dV}{dt} = 20\ m^3/s$

Find $\frac{dh}{dt}$ when $h = 2$ m.

From tank dimensions, we know $r = h$.

So $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3$

$$\frac{dV}{dt} = 3\pi h^2 \frac{dh}{dt}$$

Plug in:

$$20 = 3\pi (2)^2 \frac{dh}{dt} = 12\pi \frac{dh}{dt}$$

So $\frac{dh}{dt} = \frac{20}{12\pi} = \frac{5}{3\pi}$ m/s

2. [8 points] Let $f(x) = \frac{4}{x^2}$. $= 4x^{-2}$

a. Find the linearization of $f(x)$ when $a = 1$.

$f'(x) = -8x^{-3}$

$f(1) = \frac{4}{1^2} = 4$

$f'(1) = \frac{-8}{1^3} = -8$

$y - 4 = -8(x-1)$

$\boxed{L(x) = 4 - 8(x-1)}$

b. Use the linearization from part (a) above to estimate the value of $\frac{4}{(0.9)^2}$.

$L(0.9) = 4 - 8(0.9-1) = 4 - 8(-0.1)$

$= 4 + 0.8 = \underline{\underline{4.8}}$

3. [9 points] Let $f(x) = x^3 + 6x^2 - 15x$.

a. Find all critical points of $f(x)$.

$f'(x) = 3x^2 + 12x - 15$

$= 3(x^2 + 4x - 5)$

$= 3(x+5)(x-1) = 0$

crit. pts:

$x = 1, x = -5$

b. Determine the absolute maximum and absolute minimum of $f(x)$ on the interval $[0, 2]$. To earn full credit you must directly answer the question by stating explicitly "The maximum is .... The minimum is. ..."

x	$f(x)$
0	0
1	$1^3 + 6\cdot 1^2 - 15(1) = 1 + 6 - 15 = -8$
2	$2^3 + 6\cdot 2^2 - 15\cdot 2 = 8 + 24 - 30 = 32 - 30 = 2$

The minimum is -8.

The maximum is 2