

Name: _____

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] The following questions concern the function $f(x) = x^4 - 2x^3$. Note that the first and second derivatives of $f(x)$ are provided.

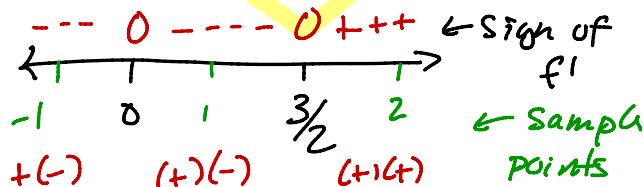
$$f'(x) = 4x^3 - 6x^2; \quad f''(x) = 12x^2 - 12x$$

$$= 2x^2(2x-3) \quad = 12x(x-1)$$

- a. Identify all critical points of $f(x)$.

Set $2x^2(2x-3) = 0$
 So $x = 0$ or $x = \frac{3}{2}$

- b. Determine intervals where $f(x)$ is increasing or decreasing.



$f(x)$ is increasing on $(\frac{3}{2}, \infty)$
 decreasing on $(-\infty, \frac{3}{2})$

- c. Identify the location (x -values) of any local maxima or minima of $f(x)$.

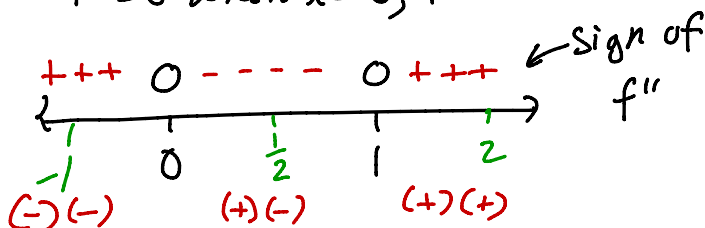
$f(x)$ has a local min at $x = \frac{3}{2}$
 $f(x)$ has no local max.

- d. Determine intervals where $f(x)$ is concave up and concave down.

$$f''(x) = 12x(x-1)$$

$$f'' = 0 \text{ when } x = 0, 1$$

$f(x)$ is concave up on $(-\infty, 0) \cup (1, \infty)$
 concave down on $(0, 1)$



- e. Identify the x -values of any inflection points of $f(x)$ or state that none exist.

$f(x)$ has inflection points at $x = 0$, and $x = 1$.

2. [8 points] Evaluate the limits below. You must justify your answer algebraically to receive full credit.

$$\text{a. } \lim_{x \rightarrow -\infty} \frac{4x^3 - x + 5}{12 - 3x - 7x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow -\infty} \frac{4 - \frac{1}{x^2} + \frac{5}{x^3}}{\frac{12}{x^3} - \frac{3}{x^2} - 7} = \frac{-4}{-7}$$

$$\text{b. } \lim_{x \rightarrow \infty} \frac{2x + 2}{\sqrt{7x^2 + 4}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{2 + \frac{2}{x}}{\sqrt{7 + \frac{4}{x^2}}} = \frac{2}{\sqrt{7}}$$

3. [5 points] Let $f(x) = \frac{4x^2 - 15}{15x^2}$.

a. Identify any vertical asymptotes and justify your answer using limits.

V.a. : $x = 0$ ← answer

$$\lim_{x \rightarrow 0^+} \frac{4x^2 - 15}{15x^2} = -\infty \quad \leftarrow \text{justification}$$

b. Identify any horizontal asymptotes justify your answer using limits.

$$\lim_{x \rightarrow +\infty} \frac{4x^2 - 15}{15x^2} = \frac{4}{15} \quad \left. \vphantom{\lim_{x \rightarrow +\infty} \frac{4x^2 - 15}{15x^2}} \right\} \text{ So } y = \frac{4}{15} \text{ is a v.a.}$$

Also check

$$\lim_{x \rightarrow -\infty} \frac{4x^2 - 15}{15x^2} = \frac{4}{15}$$

← justification ↑ answer
 ← lim as $x \rightarrow -\infty$
 adds nothing new, in this case.