Name: ——————————————————————————————   ——————— / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[12 points]** The following questions concern the function $f(x) = x^4 - 2x^3$. Note that the first and second derivatives of $f(x)$ are provided.

$$f'(x) = 4x^3 - 6x^2; \qquad f''(x) = 12x^2 - 12x.$$

   a. Identify all critical points of $f(x)$.

   b. Determine intervals where $f(x)$ is increasing or decreasing.

   c. Identify the location (x-values) of any local maxima or minima of $f(x)$ or state that none exist.

   d. Determine intervals where $f(x)$ is concave up and concave down.

   e. Identify the x-values of any inflection points of $f(x)$ or state that none exist.

2. **[8 points]** Evaluate the limits below. **You must justify your answer algebraically to receive full credit.**

    a. $\displaystyle\lim_{x\to-\infty} \frac{4x^3 - x + 5}{12 - 3x - 7x^3}$

    b. $\displaystyle\lim_{x\to\infty} \frac{2x + 2}{\sqrt{7x^2 + 4}}$

3. **[5 points]** Let $f(x) = \frac{4x^2 - 15}{15x^2}$.

    a. Identify any vertical asymptotes and **justify your answer using limits.**

    b. Identify any horizontal asymptotes **justify your answer using limits.**