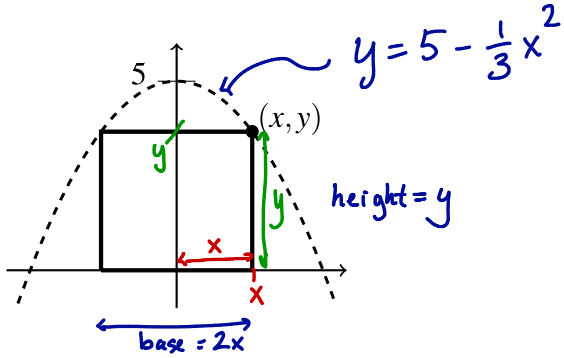
Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [8 points] (optimization) Determine the dimensions of the largest rectangle that can be inscribed in the region below the curve $y = 5 - \frac{1}{3}x^2$ and above the x-axis. Assume the base of the rectangle lies on the x axis. (See figure below.)



a. Identify the objective function. That is, identify the quantity to be maximized or minimized.

area of rectangle : $A = bh = 2xy$

b. Write the objective function as a function of x.

$A(x) = 2x\left(5 - \frac{1}{3}x^2\right) = 10x - \frac{2}{3}x^3$

c. Answer the question and use Calculus to demonstrate that you answer is correct. (That is, you need to show that you have found a minimum or maximum.)

$A'(x) = 10 - 2x^2 = 0$

$x = \pm\sqrt{5}$

(i)

The first derivative test implies A has a local maximum at $x = \sqrt{5}$. Since it is the unique critical point on $[0, \infty)$, it is an absolute maximum.

or (ii) $A''(x) = -4(x)$

$A''(\sqrt{5}) = -4\sqrt{5} < 0$,

So Second Derivative test implies that A has a local max at $x = \sqrt{5}$. Since it is the only critical point, it is an absolute max.

Height: $y = 5 - \frac{1}{3}x^2 = 5 - \frac{5}{3} = \frac{15}{3} - \frac{5}{3} = \frac{10}{3}$

Dimensions of the largest rectangle are: **base**= $2\sqrt{5}$ **height**= $\frac{10}{3}$

2. [8 points] Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

a. $\lim_{x\to 0} \frac{2e^x - 2x - 2}{3x^2} \overset{H}{=} \lim_{x\to 0} \frac{2e^x - 2}{6x} \overset{H}{=} \lim_{x\to 0} \frac{2e^x}{6} = \frac{2}{6} = \frac{1}{3}$

form $\frac{0}{0}$ form $\frac{0}{0}$

b. $\lim_{x\to 0} \frac{2x^2 - 5x}{\cos(x)} = \frac{2\cdot 0^2 - 5\cdot 0}{\cos(0)} = \frac{0}{1} = 0$

c. $\lim_{x\to 0^+} x\ln(x^4) = \lim_{x\to 0^+} \frac{4\ln(x)}{x^{-1}} \overset{H}{=} \lim_{x\to 0^+} \frac{4\cdot\frac{1}{x}}{-x^{-2}} = \lim_{x\to 0^+} \frac{-4x^2}{x} = \lim_{x\to 0^+} (-4x)$

$= -4\cdot 0 = 0$

form $0(-\infty)$ form $\frac{-\infty}{\infty}$

3. [8 points] Evaluate the following indefinite integrals. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

a. $\int (x^{1/2} + \sin(x) + 5e^x)\,dx = \frac{2}{3}x^{\frac{3}{2}} - \cos(x) + 5e^x + C$

b. $\int \left(\sec^2(x) + \frac{x+1}{x}\right) dx = \tan(x) + \int\left(1 + \frac{1}{x}\right) dx$

$= \tan(x) + x + \ln|x| + C$