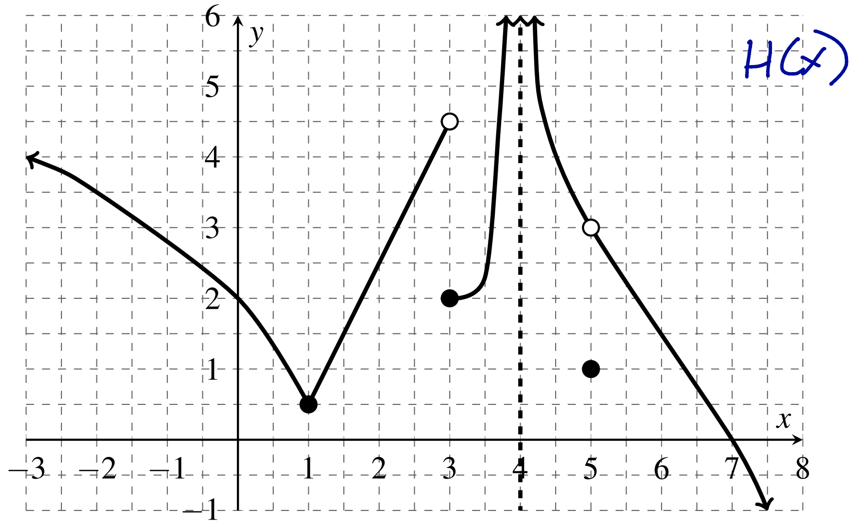
Name: ____________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

For each problem,

1. [11 points] Use the graph of the function $H(x)$ (drawn below) to answer the questions. Assume $H(x)$ has a vertical asymptote at $x = 4$. Give the most complete answer; if the limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist, write DNE.



+1 pt each

a. $f(1) =$ ½

b. $f(3) =$ 2

c. $f(5) =$ 1

d. $\lim_{x\to 3^-} f(x) =$ 4.5

e. $\lim_{x\to 3^+} f(x) =$ 2

f. $\lim_{x\to 3} f(x) =$ DNE

g. $\lim_{x\to 4} f(x) =$ $+\infty$

h. $\lim_{x\to 5} f(x) =$ 3

i. $\lim_{x\to 7} f(x) =$ 0

j. List all x-values for which the function $H(x)$ fails to be continuous.

+2pts

$x = 3, 4, 5$

2. [10 points] Evaluate the following limits. Give the most complete answer; if the limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist, write DNE. You must show work to receive full credit.

3pts

a. $\lim\limits_{x\to 4}\dfrac{2x^2-8x}{x^2-x-12} \overset{\text{plug in}}{=} \dfrac{2\cdot 4^2-8\cdot 4}{4^2-4-12} = \dfrac{32-32}{16-16} = \dfrac{0}{0}$; Try factor & cancel.

$$\lim_{x\to 4}\frac{2x(x-4)}{(x-4)(x+3)} = \lim_{x\to 4}\frac{2x}{x+3} = \frac{2\cdot 4}{4+3} = \frac{8}{7}$$

3pts

b. $\lim\limits_{x\to 1}\dfrac{\sqrt{3+x}-2}{x-1} \overset{\text{plug in}}{=} \dfrac{\sqrt{3+1}-2}{1-1} = \dfrac{0}{0}$. Try rationalizing/mult. by conjugate.

$$\lim_{x\to 1}\frac{(\sqrt{3+x}-2)}{(x-1)}\cdot\frac{(\sqrt{3+x}+2)}{(\sqrt{3+x}+2)} = \lim_{x\to 1}\frac{3+x-4}{(x-1)(\sqrt{3+x}+2)} = \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{3+x}+2)} = \lim_{x\to 1}\frac{1}{\sqrt{3+x}+2} = \frac{1}{4}$$

2pts

c. $\lim\limits_{x\to -2^+}\dfrac{5x}{x+2} \overset{\text{plug in}}{=} \dfrac{-10}{-2+2} = \dfrac{-10}{0}$ ← infinite limit. Determine sign (+ or −)

Work: As $x\to -2^+$ (#'s like $-1.9, -1.99$)

$5x \to -10$ and

$x+2 \to 0^+$

So $\dfrac{5x}{x+2} = \dfrac{-}{+} = -$

Answer: $\lim\limits_{x\to -2^+}\dfrac{\overset{-}{5x}}{\underset{+}{x+2}} = -\infty$

2pts

d. Given $\lim\limits_{x\to 10} f(x) = 5$ and $\lim\limits_{x\to 10} g(x) = -3$, evaluate $\lim\limits_{x\to 10} 2\left(\dfrac{x+1}{f(x)+g(x)}\right)$.

Plug in: $\dfrac{2(10+1)}{5-3} = \dfrac{2(11)}{2} = 11$

3. [4 points] Use the Intermediate Value Theorem to show that the polynomial $p(x) = x^3 - x + 2$ must reach a y-value of 5 for some x-value on the interval $[1,2]$.

Three observations: (a) $p(x)$ is continuous; (b) $p(1) = 2 < 5$, and (c) $p(2) = 8-2+2 = 8 > 5$.

+1 +1 +1

Conclusion: The Intermediate Value Theorem implies that the continuous function, $p(x)$, must reach every y-value between $y=2$ and $y=8$ on $[1,2]$. So $p(x)$ must reach $y=5$. +1