Name: ______________________________________ __________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [8 points] Use the limit definition of the derivative to find the derivative of $g(x) = 10 - \frac{1}{x}$. No credit will be awarded a solution that does not use the definition below.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\left(10 - \frac{1}{x+h}\right) - \left(10 - \frac{1}{x}\right)}{h} = \lim_{h \to 0} \frac{-\frac{1}{x+h} + \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{-x + x+h}{(x+h)(x)}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{h}{(x+h)(x)}\right) = \lim_{h \to 0} \frac{1}{(x+h)(x)}$$

$$= \frac{1}{(x+0)(x)} = \frac{1}{x^2}$$

2. [6 points] The distance in feet that a remote controlled car moves along a straight sidewalk is modeled by the function $s(t) = 5t^2 + t$, where t is measured in seconds after the car begins moving.

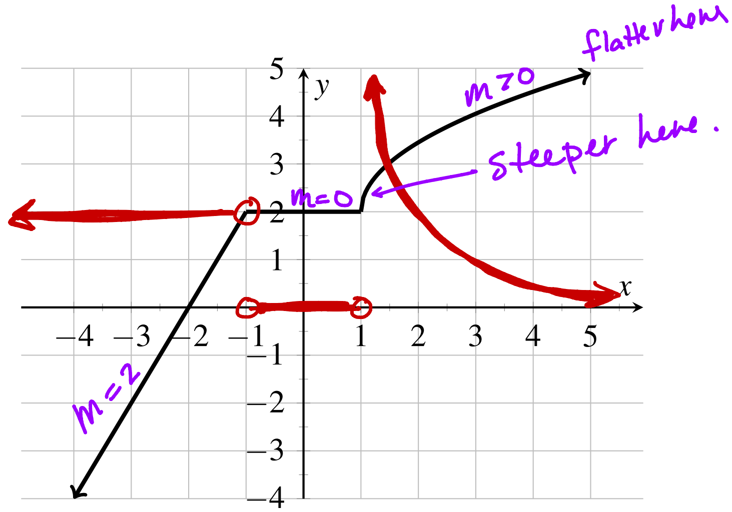
a. Find the average velocity of the car over the time interval from $t = 1$ to $t = 3$. Include units with your answer.

$$\text{average velocity} = \frac{\Delta s}{\Delta t} = \frac{s(3) - s(1)}{3-1} = \frac{(5 \cdot 3^2 + 3) - (5 \cdot 1^2 + 1)}{2} = \frac{48 - 6}{2} = 21 \text{ ft/s}$$

b. Find the instantaneous velocity of the car when $t = 1$. Include units with your answer.

$$s'(t) = v(t) = 10t + 1 \;;\quad v(1) = 10 \cdot 1 + 1 = 11 \text{ ft/s}$$

3. [5 points] The graph of $f(x)$ is below. On the same set of axes, make a rough sketch of the graph of $f'(x)$.



4. [6 points points] Find the derivative for each function below. You do not need to simplify.

a. $g(x) = 4\cos(x) + \frac{9}{x^2} + \sqrt{x} + 2$ $= 4\cos(x) + 9x^{-2} + x^{1/2} + 2$

$g'(x) = -4\sin(x) + 9(-2x^{-3}) + \frac{1}{2}x^{-1/2} + 0$

$= 4\sin(x) - 18x^{-3} + \frac{1}{2}x^{-1/2}$

b. $f(x) = \sqrt{x}(x^2+1)$ $= x^{\frac{1}{2}}(x^2+1) = x^{\frac{5}{2}} + x^{\frac{1}{2}}$

f′ using product rule

$f'(x) = \frac{1}{2}x^{-1/2}(x^2+1) + x^{1/2}(2x)$

$= \frac{x^2+1}{2\sqrt{x}} + 2x^{3/2}$

Note these are the same

f′ after distributing

$f'(x) = \frac{5}{2}x^{3/2} + \frac{1}{2}x^{-1/2}$

$= \frac{x^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} + 2x^{3/2}$

$= \frac{1}{2}x^{3/2} + \frac{1}{2}x^{-1/2} + 2x^{3/2}$

$= \frac{5}{2}x^{3/2} + \frac{1}{2}x^{-1/2}$