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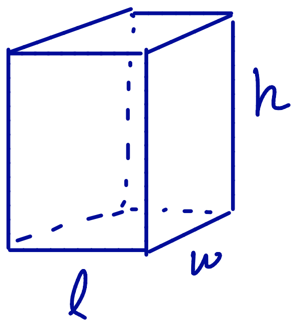
There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [9 points] A rectangular solid has constant length of $5\,m$. Its height is increasing at a rate of $2\,m/s$ and its width is decreasing at a rate of $3\,m/s$. How fast is the volume of the solid changing when the height is $9\,m$ and the width is $6\,m$.



$V = lwh$

$l = 5$

$\frac{dh}{dt} = 2$

$\frac{dw}{dt} = -3$

Find $\frac{dV}{dt}$ when $h=9$ and $w=6$

$V = 5wh$

$\frac{dV}{dt} = 5\left(w \cdot \frac{dh}{dt} + \frac{dw}{dt} h\right) = 5\left(6(2) + (-3)(9)\right)$

$= 5(12 - 27) = 5(-15) = -75\ m/s$

The volume of the solid is decreasing at a rate of 75 meters per second.

2. [8 points] Let $h(x) = x + 3e^{2x}$.

a. Find the differential of $h(x)$.

$h'(x) = 1 + 6e^{2x}$

differential: $dh = (1 + 6e^{2x})\,dx$

b. Find the linear approximation of $h(x)$ at $x = 0$.

$h(0) = 0 + 3e^{0} = +3$

$h'(0) = 1 + 6e^{0} = 7$

$y - 3 = 7(x - 0)$

Ans: $y = 3 + 7x$

c. If x changes from $x = 0$ to $x = 0.1$, estimate how much you expect $h(x)$ to change? Your answer should be a decimal or simplified fraction.

At $x=0$ and $dx = 0.1$, $dh = (1+6e^{0})(0.1) = 0.7$.

So we expect $h(x)$ to increase by 0.7.

3. [8 points] Let $f(x) = x^2(3 - 4x)$. $= 3x^2 - 4x^3$

a. Find all critical points for $f(x)$.

$f'(x) = 6x - 12x^2 = 6x(1 - 2x) = 0$

$x = 0$ or $x = \frac{1}{2}$

b. Determine the absolute maximum and absolute minimum of $f(x)$ on the interval $[-1, 1]$ or state that none exist. You must show your work to receive full credit. See the answer-blank below.

x	$f(x)$
-1	$3 + 4 = 7$
1	$3 - 4 = -1$
0	0
$\frac{1}{2}$	$\frac{1}{4}(3-2) = \frac{1}{4}$

maximum value of $f(x)$: 7

minimum value of $f(x)$: -1