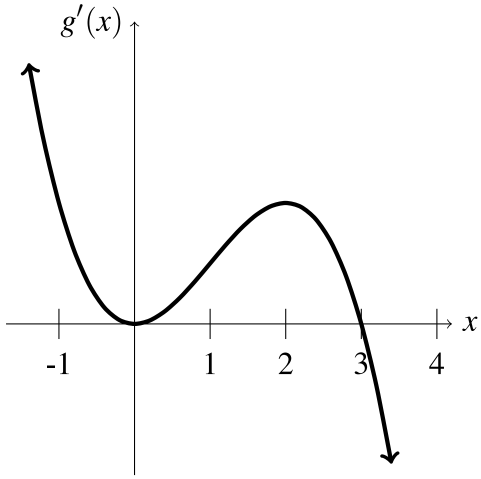
Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [8 points]

Use the graph of the **derivative** of $g(x)$, namely $g'(x)$, (below) to answer the questions about the function $g(x)$.



a. Determine the critical numbers of $g(x)$.

$x=0, \quad x=3$

(where $g'=0$)

b. Determine the intervals on which $g(x)$ is increasing and intervals on which $g(x)$ is decreasing.

$g(x)$ is $\uparrow$ on $(-\infty,3)$

$g(x)$ is $\downarrow$ on $(3,\infty)$

c. Identify the locations (x-values) of any extrema of $g(x)$. State the type of extrema (local/absolute maximum/minimum).

$g(x)$ has an absolute max at $x=3$ and no minima (absolute or local)

d. Determine the intervals on which $g(x)$ is concave up and intervals on which $g(x)$ is concave down.

$g(x)$ is c cup on $(0,2)$

$g(x)$ is ccdown on $(-\infty,0)\cup(2,\infty)$

2. [6 points] Let $H(x) = \frac{2x+1}{x-9}$

a. Identify all vertical asymptotes or state that none exist. Justify your conclusion using limits.

$x=9$

$$\lim_{x\to 9^+} \frac{2x+1}{x-9} = +\infty$$

b. Identify all horizontal asymptotes or state that none exist. Justify your conclusion using limits.

$y=2$

$$\lim_{x\to\pm\infty} \frac{2x+1}{x-9} = \lim_{x\to\pm\infty} \frac{2+\frac{1}{x}}{1-\frac{9}{x}} = 2$$

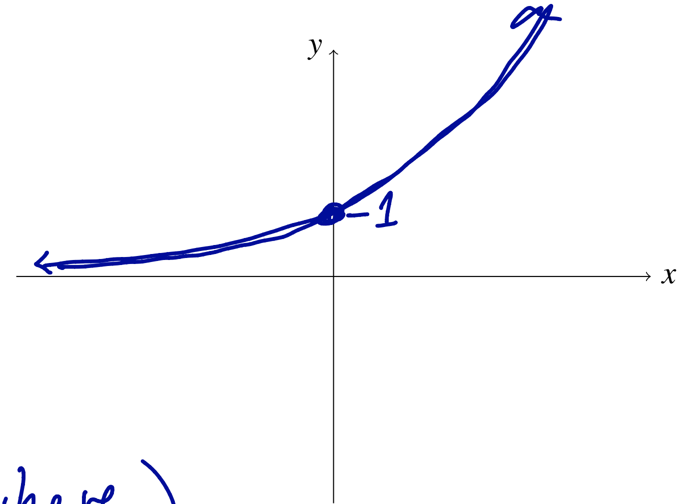
3. [3 points] On the axes below, sketch a graph of $f(x)$ that satisfies all of the properties below:

(i) $f(0) = 1$
(ii) $f'(x) > 0$ on $(-\infty, \infty)$
(iii) $f''(x) > 0$ on $(-\infty, \infty)$

f is ↑

f is ccup

(Many correct answers here.)



4. [8 points] Evaluate the limits below. Use algebra to justify your answer.

a. $\lim\limits_{x\to-\infty} \dfrac{x^2+1}{x^2-2x^3} = \lim\limits_{x\to-\infty} \dfrac{\frac{1}{x}+\frac{1}{x^3}}{\frac{1}{x}-2} = \dfrac{0}{-2} = 0$

b. $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x^4+x}}{1+x^2} = \lim\limits_{x\to\infty} \dfrac{\sqrt{2+\frac{1}{x^3}}}{\frac{1}{x^2}+1} = \sqrt{2}$

divide by
$x^2 = \sqrt{x^4}$