Name: ___Solutions___   _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (8 points) **Use the definition of the derivative** (provided below) to find the derivative of the function $f(x) = \dfrac{2}{3x}$. No credit will be awarded for finding the derivative via other methods.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{3(x+h)} - \frac{2}{3x}}{h} = \lim_{h \to 0} \left(\frac{1}{h}\right)\left(\frac{6x - 6(x+h)}{3(x+h)(3x)}\right)$$

$$= \lim_{h \to 0} \left(\frac{1}{h}\right)\left(\frac{-6h}{9(x+h)(x)}\right) = \lim_{h \to 0} \frac{-6}{9(x+h)(x)}$$

$$= \frac{-6}{9(x+0)(x)} = \boxed{\frac{-2}{3x^2}}$$

2. (4 points) The function $H(x)$ is graphed below. Sketch the graph of $H'(x)$, the derivative of $H(x)$, on the same set of axes.
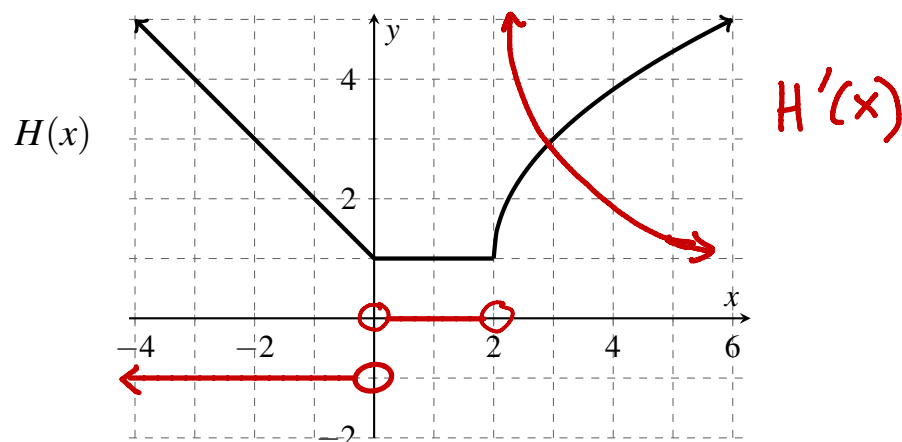
3. (9 points) Find $f'(x)$ for each function below. You do not need to simplify your answer.

(a) $f(x) = 8x^3 - 2\sqrt{x} + \sqrt{3}$ $= 8x^3 - 2x^{1/2} + \sqrt{3}$

$$f'(x) = 8 \cdot 3x^2 - 2\left(\frac{1}{2}\right)x^{-1/2} + 0$$

$$= 24x^2 - x^{-1/2}$$

(b) $f(x) = (x+1)\cos(x)$ (product rule)

$$f'(x) = (x+1)(-\sin(x)) + (1)\cos(x)$$

$$= -(x+1)\sin(x) + \cos(x)$$

(c) $f(x) = \frac{\sin(x)}{5x-4}$ (quotient rule)

$$f'(x) = \frac{(5x-4)(\cos(x)) - \sin(x)(5)}{(5x-4)^2} = \frac{(5x-4)\cos(x) - 5\sin(x)}{(5x-4)^2}$$

4. (4 points) The function $F(t)$ models the temperature in degrees Celsius of a cabin t minutes after a wood stove has been lit.

(a) Interpret $F(20) = 5$ in the context of the problem.

Twenty minutes after the stove is lit, the temperature in the cabin is 5°C.

(b) Interpret $F(20) = 1$ in the context of the problem.

Twenty minutes after the stove is lit, the temperature in the cabin is increasing at a rate of 1 °C per minute.