Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[5 points]** Find the derivatives.

   a. $G(x) = \int_0^x \sqrt{1+2t^2}\,dt$

   $$G'(x) = \sqrt{1+2x^2}$$

   b. $H(x) = \int_1^{x^3} 8\sin\left(\frac{1}{t}\right)\,dt$

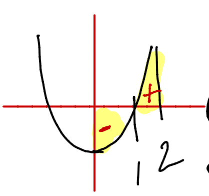
   $$H'(x) = 8\sin\left(\frac{1}{x^3}\right)\cdot(3x^2)$$
   $$= 24x^2\sin(x^{-3})$$

2. **[6 points]** The velocity of a particle moving along a straight line is given by $v(t) = t^2 - 1$ where $0 \leq t \leq 2$ is measured in seconds and v is measured in meters per second.

   a. Find the **displacement** of the particle between $t = 0$ and $t = 2$.

   $$\int_0^2 (t^2-1)\,dt = \frac{1}{3}t^3 - t\Big]_0^2 = \left(\frac{1}{3}2^3 - 2\right) - (0) = \frac{8}{3} - \frac{6}{3} = \frac{2}{3}\text{ m}$$

   b. Find the **distance traveled** of the particle between $t = 0$ and $t = 2$.

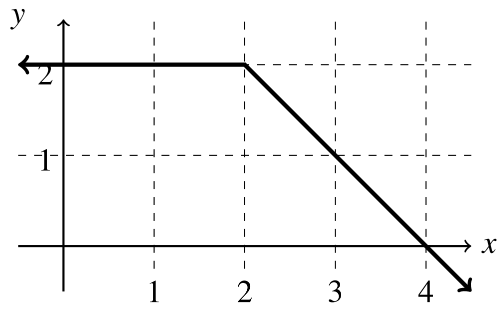


   $$\int_0^2 |v(t)|\,dt = -\int_0^1 (t^2-1)\,dt + \int_1^2 (t^2-1)\,dt = -\left[\frac{1}{3}t^3 - t\right]_0^1 + \left[\frac{1}{3}t^3 - t\right]_1^2$$
   $$= -\left(\left(\frac{1}{3}-1\right) - 0\right) + \left(\left(\frac{8}{3}-2\right) - \left(\frac{1}{3}-1\right)\right) = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{6}{3} = 2\text{ m}$$

   c. Does the problem contain sufficient information to determine the position of the particle at time $t = 2$? If so, determine the position. If not, explain why not.

   No. There is not sufficient information. There are many different position functions with the same velocity function. We need to know the position at some time t.

3. [4 points] Use the graph of $f(x)$ (below) to answer questions about $A(x) = \int_0^x f(t)\,dt$.



a. $A(0) =$ $\int_0^0 f(t)\,dt = 0$

b. $A(4) =$ $\int_0^4 f(t)\,dt = 6$

c. At $x = 3$, is $A(x)$ increasing, decreasing, or neither?

increasing. (because $f(t) > 0$)

4. [10 points] Evaluate the definite integrals below.

a. $\int_1^3 (2 - 6x^2)\,dx = 2x - 2x^3 \Big]_1^3 = (2\cdot 3 - 2\cdot 3^3) - (2\cdot 1 - 2\cdot 1^3)$

$= (6 - 54) - (2 - 2) = -48$

b. $\int_0^1 \sin(5x)\,dx = -\frac{1}{5}\cos(5x)\Big]_0^1 = -\frac{1}{5}\left(\cos(5) - \cos(0)\right)$

$= -\frac{1}{5}\left(\cos(5) - 1\right) = \frac{1}{5}\left(1 - \cos(5)\right)$

c. $\int_0^2 \frac{x^2}{\sqrt{1+x^3}}\,dx = \int_1^9 \frac{\frac{1}{3}du}{\sqrt{u}} = \frac{1}{3}\int_1^9 u^{-1/2}\,du = \frac{1}{3}\cdot\frac{2}{1}\,u^{\frac{1}{2}}\Big]_1^9$

let $u = 1 + x^3$
$du = 3x^2\,dx$
$x = 0, u = 1$
$x = 2, u = 9$

$= \frac{2}{3}\left(9^{1/2} - 1^{1/2}\right) = \frac{2}{3}(3 - 1) = \frac{4}{3}$