Name: Solutions ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [10 points] On a given day, the flow rate F, measured in cars per hour, on a congested roadway is given by $F(v) = \dfrac{5v}{v^2+100}$ where v is the speed of the traffic in miles per hour.

a. Given the context of the problem, what is a reasonable domain?

D: $[0, \infty)$ or D: $[0, 1000]$ some number well beyond what cars can do.

b. Find all critical numbers of the $F(v)$ in your domain in part (a).

$$F'(v) = \frac{(v^2+100)(5) - 5v(2v)}{(v^2+100)^2} = \frac{5(v^2+100-2v^2)}{(v^2+100)^2} = \frac{5(100-v^2)}{(v^2+100)^2}$$

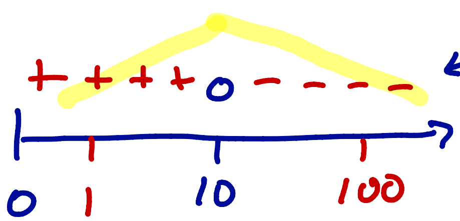
F' is never undefined.

$F' = 0$ when $100 - v^2 = 0$ or $v = \pm 10$.

But only $\boxed{v = +10}$ lies in our domain

c. **Use Calculus** to determine if the critical number(s) correspond to an absolute minimum or absolute maximum or neither. Note providing organized clear work here is crucial.

Use First Derivate Test



← sign of F'

$$F'(1) = \frac{5(100)}{(1+100)^2} > 0$$

$$F'(100) = \frac{5(100-100^2)}{(100^2+100)^2} < 0$$

Conclusion: F has an absolute maximum at $V=10$

d. Write a complete sentence explaining what your work in part (c) indicates about traffic flow.

The flow of traffic is maximized when the speed of the traffic is 10 mph.

2. [8 points] Evaluate the following limits. Show your work.

a. $\lim_{x\to 0} \frac{2(e^x - x - 1)}{3x^2} \overset{H}{=} \lim_{x\to 0} \frac{2(e^x - 1)}{6x} \overset{H}{=} \lim_{x\to 0} \frac{2e^x}{6} = \frac{1}{3}$

b. $\lim_{x\to 0^+} x^x = \boxed{e^0 = 1}$

$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1}} \overset{H}{=} \lim_{x\to 0^+} \frac{x^{-1}}{-x^{-2}} = \lim_{x\to 0^+} -x = 0$

3. [7 points] Evaluate the following indefinite integrals. Show your work.

a. $\int (x^{2/3} + x^{-1/3})dx = \frac{x^{5/3}}{\frac{5}{3}} + \frac{x^{2/3}}{\frac{2}{3}} + C = \frac{3}{5}x^{5/3} + \frac{3}{2}x^{2/3} + C$

b. $\int (\sec^2(x) - \frac{x^2+1}{x^2})dx = \int (\sec^2(x) - 1 - x^{-2})dx$

$= \tan(x) - x + x^{-1} + C$