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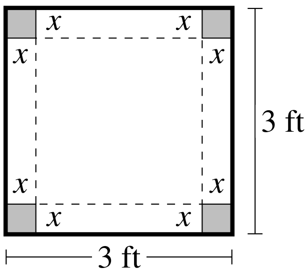
There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [11 points] An open box is to be constructed by cutting squares out of the four corners of a 3 foot by 3 foot piece of cardboard and folding up the sides. (See the diagram. Note that the box will not have a lid, and the height of the box will be x feet.)



a. Write an equation for the **volume** of the box in terms of the variable x.

$$V = l \cdot w \cdot h = (3-2x)(3-2x)x$$
$$= (3x-2x^2)(3-2x)$$
$$= 9x - 6x^2 - 6x^2 + 4x^3$$

b. Determine the **dimensions** of the box with the largest volume. Show your work, and use calculus to **justify** that your answer is the maximum. Include units in your final answer. An answer with no clear justification will not receive full credit.

$$V(x) = 4x^3 - 12x^2 + 9x$$
$$V'(x) = 12x^2 - 24x + 9$$
$$V'(x) = 0 \Rightarrow 12x^2 - 24x + 9 = 0$$
$$\Rightarrow 4x^2 - 8x + 3 = 0$$
$$\Rightarrow (2x-1)(2x-3) = 0$$
$$\Rightarrow x = 1/2 \text{ or } x = 3/2$$

Domain: $[0, 3/2]$ (given context)

Method #1: extreme value theorem

$V(0) = 0$, $V(3/2) = (3-3)(3-3)(3/2) = 0$

$V(1/2) = (3-1)(3-1)(1/2) = 2 \leftarrow$ MAX

Method #2: 2nd deriv. test

$V''(x) = 24x - 24$

$V''(1/2) = 12 - 24 < 0$ ∩

so $x = 1/2$ is the only max on the domain.

length $= 3 - 2(1/2) = 2 =$ width

height $= x = 1/2$

Dimensions: length: 2 width: 2 height: 1/2

2. [8 points] Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

a. $\lim_{x\to 0} \frac{3e^x - 3x - 3}{x^2}$

$e^0 = 1$, so DS: $\frac{3\cdot 1 - 0 - 3}{0} = 0/0^+$

$\overset{L'H}{=} \lim_{x\to 0} \frac{3e^x - 3}{2x}$ type $0/0$

$\overset{L'H}{=} \lim_{x\to 0} \frac{3e^x}{2} = \frac{3}{2}$

b. $\lim_{x\to+\infty} x \sin\left(\frac{1}{x}\right)$ type $\infty \cdot 0$

$= \lim_{x\to\infty} \frac{\sin(1/x)}{1/x}$ type $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\cos(1/x)(-x^{-2})}{-x^{-2}}$

$= \lim_{x\to\infty} \cos(1/x) = 1$

3. [6 points] Evaluate the following indefinite integrals.

a. $\int (x^{3/2} + \sin(x) + 5e^x)\, dx$

$= \frac{x^{5/2}}{5/2} + (-\cos(x)) + 5e^x + C.$

Check:
$\frac{d}{dx}\left(\frac{2}{5}x^{5/2} - \cos x + 5e^x + C\right)$
$= \frac{2}{5}\left(\frac{5}{2}\right)x^{3/2} + \sin(x) + 5e^x$ ✓

b. $\int \left(\sec^2(x) + \frac{x+1}{x}\right) dx = \int \sec^2(x) + 1 + 1/x \, dx$

$= \tan(x) + x + \ln|x| + C$

Check:
$\frac{d}{dx}(\tan x + x + \ln|x| + C) =$
$\sec^2 x + 1 + \frac{1}{x}.$