Name: Key ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[2 points]** Use the Fundamental Theorem of Calculus to evaluate the following derivative:

$$\frac{d}{dx}\left(\int_x^7 \sqrt{|t+8|}\,dt\right)$$

$$= \frac{d}{dt}\left(-\int_7^x \sqrt{|t+8|}\,dt\right) = \boxed{-\sqrt{|x+8|}}$$

2. **[12 points]** Evaluate the following indefinite integrals. Show your work and state whenever you use a substitution.

a. $\int x^2(x^3-2)^2\,dx = \frac{1}{3}\int u^2\,du = \frac{1}{9}u^3 + C = \boxed{\frac{1}{9}(x^3-2)^3 + C}$

Let $u = x^3-2$, $du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

b. $\int \sin x \cos x\,dx = \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}\sin^2 x + C}$

Let $u = \sin x$, $du = \cos x\,dx$

(could also let $u = \cos x$)

c. $\int \frac{\ln x + 3}{x}\,dx = \int (u+3)\,du = \frac{1}{2}u^2 + 3u + C = \boxed{\frac{1}{2}(\ln x)^2 + 3\ln x + C}$

Let $u = \ln x$, $du = \frac{1}{x}dx$

(could also let $u = \ln x + 3$)

3. [5 points] Find the area under the curve $f(x) = x^2 + 2x + e^x$ from $x = 0$ to $x = 3$. Use a definite integral, show all your work, and **simplify your final answer**.

$$\int_0^3 (x^2 + 2x + e^x)\,dx = \left[\frac{1}{3}x^3 + x^2 + e^x\right]_0^3 = \left(\frac{1}{3}(3)^3 + (3)^2 + e^3\right) - \left(\frac{1}{3}(0)^3 + (0)^2 + e^0\right)$$

$$= 9 + 9 + e^3 - 1 = \boxed{17 + e^3}$$

4. [6 points] A ball is thrown upward from an initial height of 5 feet at an initial speed of 40 feet per second. Its upward velocity at t seconds is given by the equation $v(t) = -32t + 40$ feet per second.

a. Evaluate $\int_0^2 v(t)\,dt$.

$$\int_0^2 (-32t + 40)\,dt = \left[-16t^2 + 40t\right]_0^2$$

$$= -16(2)^2 + 40(2) - \left(-16(0)^2 + 40(0)\right)$$

$$= -64 + 80 = \boxed{16}$$

b. Explain what the quantity $\int_0^2 v(t)\,dt$ represents. Give units.

The ball is 16 feet higher than when it was thrown (after 2 seconds).

From 0 to 2 seconds, the ball's net displacement is 16 ft.

c. Explain how would this answer change if the ball had been thrown from an initial height of 10 feet?

The answer does not change. We never used the initial height in our calculation of $\int_0^2 v(t)\,dt$.