Name: Key _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[8 points]** Use the **limit definition** (given below) of the derivative to find the derivative of $f(x) = \frac{2}{x^2}$. **No credit will be awarded for a solution that does not use the definition below.** Show all your work clearly, step by step, using correct notation.

$$f'(x) := \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

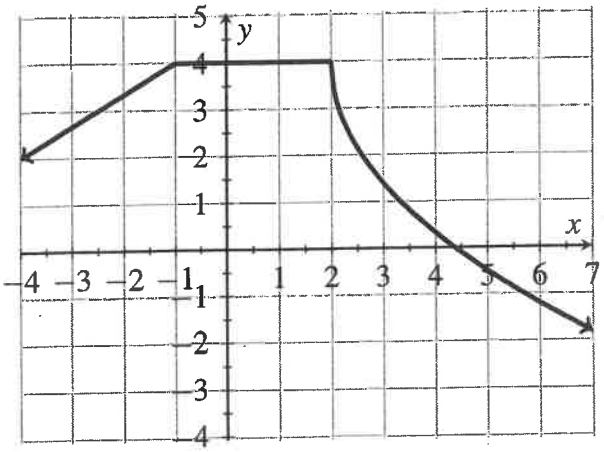
$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h} \cdot \frac{(x+h)^2x^2}{(x+h)^2x^2}$$

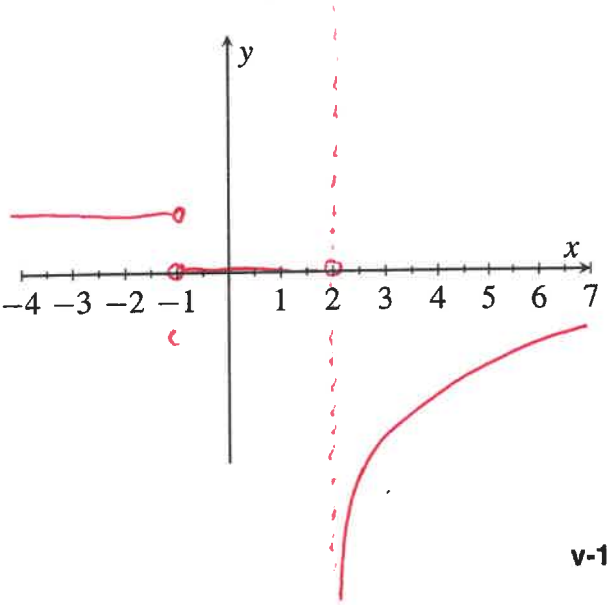
$$= \lim_{h\to 0} \frac{2x^2 - 2(x+h)^2}{h(x+h)^2x^2} = \frac{2x^2 - 2x^2 - 4xh - 2h^2}{h(x+h)^2x^2}$$

$$= \lim_{h\to 0} \frac{-4xh - 2h^2}{h(x+h)^2x^2} = \lim_{h\to 0} \frac{-4x - 2h}{(x+h)^2x^2} = \frac{-4x - 2(0)}{(x+0)^2x^2}$$

$$= \frac{-4x}{x^2 \cdot x^2} = \boxed{-\frac{4}{x^3}}$$

2. **[5 points]** The graph of $f(x)$ is shown below. On the **other set of axes**, sketch the graph of $f'(x)$. If there are any asymptotes, draw them with dashed lines. Use open circles to show points where the derivative is not defined, if any. (You are not given values on the y-axis; I am interested in the correct shape/holes/asymptotes of the derivative, not the specific values.)





3. **[6 points]** Use the derivative rules to find the derivative for each function below. **Do not simplify your answer.** Show work so someone else can follow your thinking.

   a. $f(x) = (\cos x)(\sqrt{x} - 2x)$ $= \cos x \left(x^{\frac{1}{2}} - 2x\right)$

   $f'(x) =$ $(-\sin x)(\sqrt{x} - 2x) + \cos x \left(\frac{1}{2}x^{-\frac{1}{2}} - 2\right)$

   b. $g(x) = \dfrac{x^3 - 2x + 1}{x}$ $= x^2 - 2 + x^{-1}$

   $g'(x) =$ $2x - x^{-2}$

4. **[6 points]** For the function $F(x) = x^3 - \sin x$, find $F'(x)$, $F''(x)$, and $F'''(x)$.

   $F'(x) =$ $3x^2 - \cos x$

   $F''(x) =$ $6x + \sin x$

   $F'''(x) =$ $6 + \cos x$