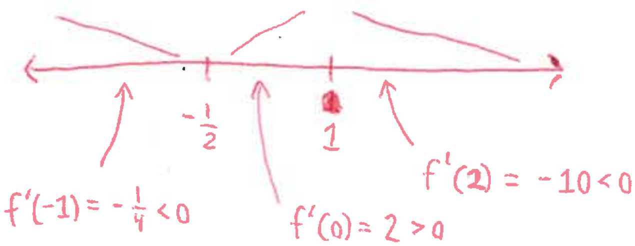
Name: Key ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] Answer the questions below about the function $f(x) = \dfrac{x^2+2x}{x^2-2x+1}$. Observe that $f'(x) = \dfrac{-4x-2}{(x-1)^3}$ and $f''(x) = \dfrac{8x+16}{(x-1)^4}$.

a. Find all intervals where f is **increasing** and where f is **decreasing**.

$f'(x)$ is undefined when $x=1$, $f'(x)$ is 0 when $-4x-2=0 \Rightarrow x = -\frac{1}{2}$



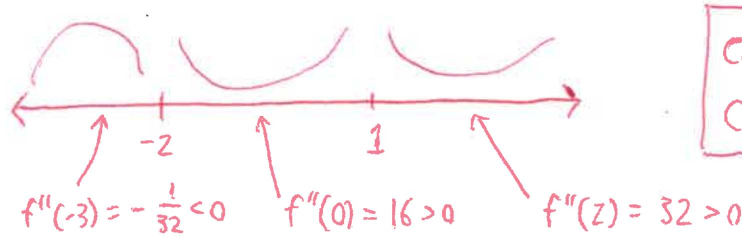
$\boxed{\text{Increasing: } (-\tfrac{1}{2}, 1) \quad \text{Decreasing: } (-\infty, -\tfrac{1}{2}) \cup (1, \infty)}$

b. Find the x-values of **all local minima and local maxima** of f or state that none exist.

There is a local min at $x = -\frac{1}{2}$.

c. Find all intervals where f is **concave up** and where f is **concave down.**

$f''(x)$ is undefined when $x=1$, $f''(x)$ is 0 when $8x+16=0 \Rightarrow x=2$



$\boxed{\text{Concave up: } (-2,1) \cup (1,\infty) \quad \text{Concave down: } (-\infty, -2)}$

d. Determine whether $f(x)$ has any **horizontal asymptotes**. Show you work.

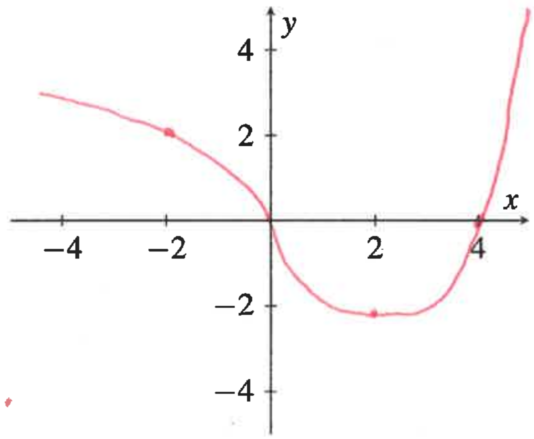
$$\lim_{x\to\infty} \frac{x^2+2x}{x^2-2x+1} = \lim_{x\to\infty} \frac{1+\frac{2}{x}}{1-\frac{2}{x}+\frac{1}{x^2}} = \frac{1}{1} = 1$$

$\boxed{\text{H.A. of } y=1}$

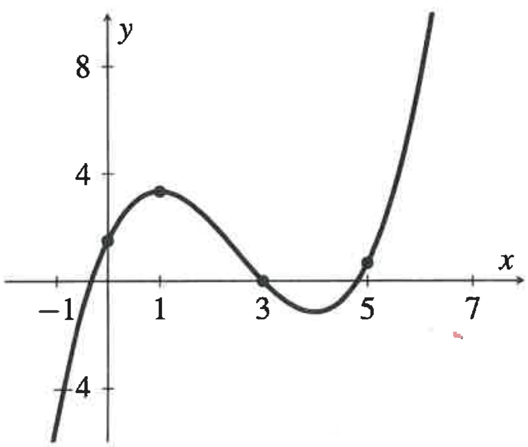
$$\lim_{x\to-\infty} \frac{x^2+2x}{x^2-2x+1} = \lim_{x\to-\infty} \frac{1+\frac{2}{x}}{1-\frac{2}{x}+\frac{1}{x^2}} = \frac{1}{1} = 1$$

2. [7 points] Sketch a graph of a function $h(x)$ that satisfies the following criteria:

- $h(-2) = 2$, $h(2) = -2$, and $h(4) = 0$
- $h'(x) < 0$ when $x < 2$
- $h'(x) > 0$ when $x > 2$
- $h''(x) < 0$ when $x < 0$
- $h''(x) > 0$ when $x > 0$



3. [6 points] Use the graph of the function $g(x)$ (below) to determine whether each value below is **positive**, **negative**, **zero**, or **undefined**. You do not need to justify your answers.



a. $g''(0)$ negative

b. $g'(1)$ zero

c. $g''(1)$ negative

d. $g'(3)$ negative

e. $g'(5)$ positive

f. $g''(5)$ positive