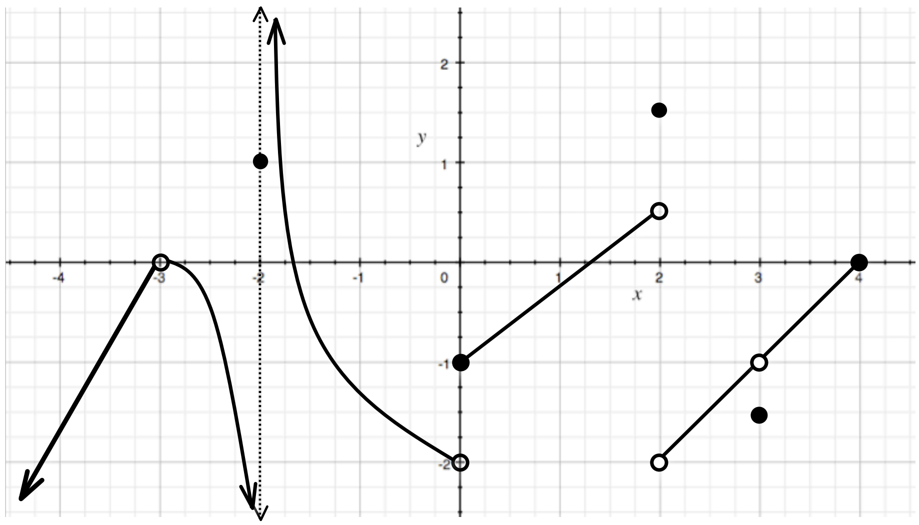
Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] Consider the graph of the function f below.



Use the graph of f to answer each question below. If the limit is infinite, indicate that with ∞ or $-\infty$. If the value does not exist or is undefined, write DNE.

(a) $\lim_{x\to -3} f(x) =$ 0

(b) $\lim_{x\to -2} f(x) =$ DNE

(c) $\lim_{x\to 1} f(x) =$ $-\frac{1}{4}$

(d) $\lim_{x\to -2^+} f(x) =$ ∞

(e) $\lim_{x\to 2^-} f(x) =$ $\frac{1}{2}$

(f) $\lim_{x\to 3^-} f(x) =$ -1

(g) $f(-3) =$ DNE

(h) $f(2) =$ 1.5

(i) $f(3) =$ -1.5

(j) Indicate **all** x-values for which the function f is **not continuous**.

$x = -3, -2, 0, 2, 3$

2. **[9 points]** Evaluate the following limits. Justify your answers.

a. $\lim_{x\to 2} x^2 - 3x + 5$

$= 2^2 - 3(2) + 5$ since $x^2 - 3x + 5$ is continuous

$= 3$

b. $\lim_{x\to 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4}$

$= \lim_{x \to 4} \frac{(x-4)(x+2)}{(x-4)(x-1)} = \lim_{x \to 4} \frac{x+2}{x-1}$

$= \frac{4+2}{4-1}$ by continuity

$= 2$

c. $\lim_{\theta\to\pi} \frac{\tan\theta}{\sin\theta}$ $\left(\text{Hint: } \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \lim_{\theta \to \pi} \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\sin\theta} = \lim_{\theta \to \pi} \frac{1}{\cos\theta}$

$= \frac{1}{-1}$ by continuity

$= -1$

3. [4 points] Determine whether or not the given function is continuous at $x = 5$. **Justify your answer using limits.**

$$f(x) = \begin{cases} \frac{x+3}{x-1} & \text{if } x < 5 \\ x^2 - 3x - 8 & \text{if } x \geq 5 \end{cases}$$

$\lim_{x \to 5^-} f(x) = \frac{5+3}{5-1} = 2$

$\lim_{x \to 5^+} f(x) = 5^2 - 3(5) - 8 = 2$

So $\lim_{x \to 5} f(x) = 2$.

$f(5) = 2$.

So $\lim_{x \to 5} f(x) = f(5)$ and f is continuous at $x = 5$

4. [2 points] BONUS: Does the equation $2x^7 - x^5 = 3x^{31} + 5x^{13} + 2x^7 + x^3$ have a solution on the interval $[-1, 1]$? Justify your answer.

Let $f(x) = (3x^{31} + 5x^{13} + 2x^7 + x^3) - (2x^2 - x^5)$

$f(x)$ is continuous

$f(-1) = 10$, $f(1) = 10$

So by the IVT, there is a c in $(0,0)$ such that $f(c) = 0$,

So c is a solution to the above equation in $[1, 1]$