

Name: Key

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] Find the derivative of each function. You do not need to simplify your answers.

a. $y = 5 \cot(3x^2)$

$$\frac{dy}{dx} = 5(-\csc^2(3x^2) \cdot 6x) = \boxed{-30 \csc^2(3x^2) \cdot x}$$

b. $f(\theta) = \sqrt{\tan\left(\frac{\pi}{4}\right) - \cos(\theta)}$

$$f'(\theta) = \boxed{\frac{1}{2} \left(\tan\left(\frac{\pi}{4}\right) - \cos\theta \right)^{-\frac{1}{2}} \cdot \sin\theta}$$

$$\text{c. } g(\alpha) = \frac{\tan(\alpha) \csc(\alpha)}{\sec(\alpha)} = \frac{\frac{\sin(\alpha)}{\cos(\alpha)} \cdot \frac{1}{\sin(\alpha)}}{\frac{1}{\cos(\alpha)}} = \frac{\frac{1}{\cos(\alpha)}}{\frac{1}{\cos(\alpha)}} = 1$$

$$g'(\alpha) = \boxed{0}$$

d. $h(x) = \sin^5(x^2 - 3x + 8)$

$$h'(x) = \boxed{5 \sin^4(x^2 - 3x + 8) \cdot (2x - 3)}$$

2. [5 points] Find all x -values where the graph of $f(x) = (x^2 - 8x)^4$ has a horizontal tangent line.

$$f'(x) = 4(x^2 - 8x)^3 \cdot (2x - 8)$$

$$\text{Set } f'(x) \text{ equal to } 0: 4(x^2 - 8x)^3(2x - 8) = 0 \Rightarrow x^2 - 8x = 0 \text{ or } 2x - 8 = 0$$

$$\Rightarrow x(x - 8) = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow x = 0, 4, 8$$

f has a horizontal tangent at $x = 0, 4, 8$.

3. [8 points] Find $f'''(x)$ if $f(x) = 3x \sin(2x + 5)$.

$$\begin{aligned} f'(x) &= 3 \sin(2x + 5) + 3x \cos(2x + 5) \cdot 2 \\ &= 3 \sin(2x + 5) + 6x \cos(2x + 5) \end{aligned}$$

$$\begin{aligned} f''(x) &= 3 \cos(2x + 5) \cdot 2 + 6 \cos(2x + 5) - 6x \sin(2x + 5) \cdot 2 \\ &= 12 \cos(2x + 5) - 12x \sin(2x + 5) \end{aligned}$$

$$\begin{aligned} f'''(x) &= -12 \sin(2x + 5) \cdot 2 - 12 \sin(2x + 5) - 12x \cos(2x + 5) \cdot 2 \\ &= \boxed{-24 \sin(2x + 5) - 24x \cos(2x + 5)} \end{aligned}$$