Name: ______________________ ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[12 points]** Find the derivative of each function. You do not need to simplify your answers.

   a. $y = 3x^3 + 4e^x - 5\ln(3)$

   $\frac{dy}{dx} = \boxed{9x^2 + 4e^x + 0}$

   b. $f(x) = \arcsin\left(x^3\right)$

   $f'(x) = \boxed{\frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2}$

   c. $g(x) = \ln\left(\frac{x^5}{\cos(x)}\right)$ $= \ln(x^5) - \ln(\cos x) = 5\ln x - \ln(\cos x)$

   $g'(x) = \frac{5}{x} - \frac{1}{\cos x} \cdot (-\sin x) = \boxed{\frac{5}{x} + \tan x}$

   d. $h(x) = 5\sec(e^x) + \ln(e^x)$ $= 5\sec(e^x) + x$

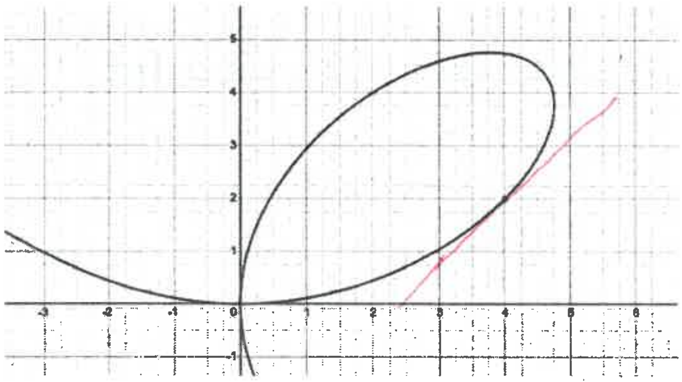
   $h'(x) = \boxed{5\sec(e^x)\tan(e^x) \cdot e^x + 1}$

2. **[4 points]** Determine if the functions $f(x) = \ln(2x)$ and $g(x) = \ln(3x)$ have the same derivative. Justify your answer.

   $f'(x) = \frac{1}{2x} \cdot 2 = \frac{1}{x}$ $g'(x) = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

   Yes, they have the same derivative.

3. [9 points] The graph of $x^3 + y^3 = 9xy$ is given below.



a. Calculate $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[x^3+y^3\right] = \frac{d}{dx}\left[9xy\right]$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 9y + 9x \cdot \frac{dy}{dx}$$

$$3y^2 \frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{9y-3x^2}{3y^2-9x}}$$

b. Use $\frac{dy}{dx}$ to find the **slope** of the tangent line to the curve at $(4,2)$. **Simplify** your answer.

$$\left.\frac{dy}{dx}\right|_{(4,2)} = \frac{9\cdot 2 - 3\cdot 16}{3\cdot 4 - 9\cdot 4} = \frac{18-48}{12-36} = \frac{-30}{-24} = \boxed{\frac{5}{4}}$$

4. [2 points] BONUS: Given the function $f(x) = (\arctan x)^x$, find $f'(x)$. Let $y = f(x)$

$$\ln y = \ln\left((\arctan x)^x\right) = x \ln(\arctan x)$$

$\frac{d}{dx}$ both sides

$$\frac{1}{y}\cdot\frac{dy}{dx} = \ln(\arctan x) + x \cdot \frac{1}{\arctan x}\cdot\frac{1}{1+x^2}$$

$$\frac{dy}{dx} = y\left(\ln(\arctan x) + \frac{x}{(1+x^2)\arctan x}\right) = \boxed{(\arctan x)^x\left[\ln(\arctan x) + \frac{x}{(1+x^2)\arctan x}\right]}$$