

Name: _____

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [9 points] You and your friend leave campus at the same time. You walk due north at 4 kilometers per hour while your friend walks due east at 6 kilometers per hour. After you have traveled 2 kilometers, at what rate is the distance between you changing?

t = time in hours

y = distance you have travelled in km

f = distance friend has travelled in km

D = distance between you and friend

$$y^2 + f^2 = D^2$$

$$\frac{d}{dt}(y^2 + f^2) = \frac{d}{dt} D^2$$

$$2y \frac{dy}{dt} + 2f \frac{df}{dt} = 2D \frac{dD}{dt}$$

We know that when you have walked 2 km, 0.5 hrs have passed, so your friend has walked 3 km, and $2^2 + 3^2 = D^2$, so $D = \sqrt{13}$.

Moreover, $\frac{dy}{dt} = 4$, $\frac{df}{dt} = 6$. So we have

$$2(2)(4) + 2(3)(6) = 2\sqrt{13} \frac{dD}{dt}$$

$$\text{So } \boxed{\frac{dD}{dt} = \frac{26}{\sqrt{13}} \frac{\text{km}}{\text{hr}}}$$

2. [8 points] Let $g(x) = x + 6e^{2x}$.

a. Find the linear approximation of $g(x)$ at $x = 0$.

$$\begin{aligned} g'(x) &= 1 + 12e^{2x} \\ g'(0) &= 13 \\ g(0) &= 6 \end{aligned} \quad \begin{aligned} y - 6 &= 13(x - 0) \\ y &= 13x + 6 \\ \boxed{L} &= 13x + 6 \end{aligned}$$

b. Find the differential of $g(x)$.

$$dy = (1 + 12e^{2x})dx$$

c. If x changes from $x = 0$ to $x = 0.1$, estimate how much you expect $g(x)$ to change. Express your answer as a decimal or as a simplified fraction.

$$dy = (1 + 12e^{2(0)})(0.1) = \boxed{1.3 \quad \text{or} \quad \frac{13}{10}}$$

3. [8 points] Let $h(x) = 3x^4 - 4x^3 + 1$.

a. Find all critical points of $h(x)$.

$$\begin{aligned} h'(x) &= 12x^3 - 12x^2 \\ &= 12x^2(x - 1) \end{aligned}$$

$$\begin{aligned} 12x^2(x - 1) &= 0 \\ \boxed{x = 0, x = 1} \end{aligned}$$

b. Determine the absolute maximum and absolute minimum of $h(x)$ on the interval $[-1, 2]$

x	$h(x)$
-1	8
0	1
1	0
2	17

maximum value of $h(x)$: 17
 minimum value of $h(x)$: 0