

Name: \_\_\_\_\_

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [11 points] Let  $g(x) = \frac{3}{2}x^4 - 3x^3$ . Note that we have the following:

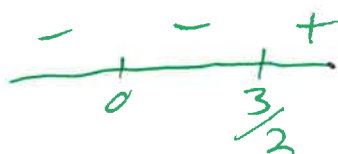
$$g'(x) = 6x^3 - 9x^2$$

$$g''(x) = 18x^2 - 18x$$

**You must show your work for all parts to receive credit!**

- a. Determine the intervals where  $g$  is **increasing** and where  $g$  is **decreasing**.

$$\begin{aligned} g'(x) &= 0 \\ 6x^3 - 9x^2 &= 0 \\ 3x^2(2x - 3) &= 0 \\ x &= 0, x = \frac{3}{2} \end{aligned}$$



decreasing:  $(-\infty, \frac{3}{2})$   
increasing:  $(\frac{3}{2}, \infty)$

- b. Find the  $x$  values where any **local maxima** occur and where any **local minima** occur.

local max: none  
local min at  $x = \frac{3}{2}$

- c. Find the intervals where  $g$  is **concave up** and where  $g$  is **concave down**.

$$\begin{aligned} g''(x) &= 0 \\ 18x^2 - 18x &= 0 \\ 18x(x - 1) &= 0 \\ x &= 0, x = 1 \end{aligned}$$



conc up:  
 $(-\infty, 0) \cup (1, \infty)$   
conc down:  
 $(0, 1)$

- d. Find the  $x$  values where any **inflection points** occur.

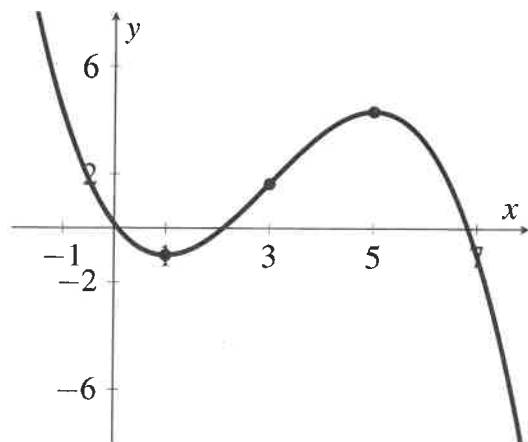
$x = 0, x = 1$

2. [8 points] Evaluate the following limits. Show your work!

$$\begin{aligned}
 \text{a. } \lim_{x \rightarrow -\infty} \frac{x^2 + 1}{x^2 - 2x^3} &= \lim_{x \rightarrow -\infty} \frac{x^2 + 1}{x^2 - 2x^3} \cdot \frac{(\frac{1}{x^3})}{(\frac{1}{x^3})} \\
 &= \lim_{x \rightarrow -\infty} \frac{\frac{x^2}{x^3} + \frac{1}{x^3}}{\frac{x^2}{x^3} - \frac{2x^3}{x^3}} \\
 &= \lim_{x \rightarrow -\infty} \frac{\frac{1}{x} + \frac{1}{x^3}}{\frac{1}{x} - 2} = \frac{0}{-2} = \boxed{0}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \lim_{x \rightarrow \infty} \frac{4x - 2}{\sqrt{5x^2 - 4}} &= \lim_{x \rightarrow \infty} \frac{4x - 2}{\sqrt{5x^2 - 4}} \cdot \frac{(\frac{1}{x})}{(\frac{1}{x})} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{4x}{x} - \frac{2}{x}}{\sqrt{\frac{5x^2}{x^2} - \frac{4}{x^2}}} \\
 &= \lim_{x \rightarrow \infty} \frac{4 - \frac{2}{x}}{\sqrt{5 - \frac{4}{x^2}}} = \boxed{\frac{4}{\sqrt{5}}}
 \end{aligned}$$

3. [6 points] Based on the graph of the function  $f(x)$  below, determine whether each value is positive, negative, zero, or undefined. You do not need to show your work.



a.  $g'(1) = 0$

b.  $g''(1) > 0$

c.  $g'(3) > 0$

d.  $g''(3) = 0$

e.  $g'(5) = 0$

f.  $g''(5) < 0$