April 3, 2025 **Math 251: Quiz 9**

Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[9 points]** Find the radius r and height h of the cylinder with surface area 24π that has the largest possible volume. The formulas for the volume V and surface area S are given below.

$V = \pi r^2 h, \qquad S = 2\pi r^2 + 2\pi rh$

a. State the value that you want to maximize/minimize.

Volume (V)

b. Write the value from part (a) as a function of a single variable.

$S = 2\pi r^2 + 2\pi rh$

$24\pi = 2\pi r^2 + 2\pi rh$

$12 = r^2 + rh$

$h = \frac{12}{r} - r$

$V = \pi r^2 h$

$V = \pi r^2 \left(\frac{12}{r} - r\right)$

$\boxed{V = 12\pi r - \pi r^3}$

c. Answer the original question and use calculus to justify your answer.

$V' = 12\pi - 3\pi r^2$

$12\pi - 3\pi r^2 = 0$

$r^2 = 4$

$r = 2$ (−2 is a solution, but doesn't make sense as a radius)

+ −

2

(second derivative test works too)

hence max, not min

If $r = 2$, then

$h = \frac{12}{2} - 2 = 4$

so $\boxed{r = 2,\ h = 4}$

2. [8 points] Use L'Hôpital's Rule to evaluate the limits below. Indicate your use of L'Hôpital's Rule with $\stackrel{h}{=}$ or $\stackrel{L'H}{=}$ or something similar. (Be sure to verify explicitly that L'Hôpital's Rule applies!).

a. $\lim\limits_{x\to 1^+} \dfrac{x+\cos(\pi)}{\ln(x)} \stackrel{LH}{=} \lim\limits_{x\to 1^+} \dfrac{1}{\left(\frac{1}{x}\right)} = \lim\limits_{x\to 1^+} x = 1$

form $\frac{0}{0}$

b. $\lim\limits_{x\to 0^+} x^x = \lim\limits_{x\to 0^+} e^{\ln(x^x)} \underset{\text{see below}}{=} e^0 = 1$

form 0^0

$\lim\limits_{x\to 0^+} \ln(x^x) = \lim\limits_{x\to 0^+} x\ln(x) = \lim\limits_{x\to 0^+} \dfrac{\ln(x)}{\left(\frac{1}{x}\right)}$ form $\frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim\limits_{x\to 0^+} \dfrac{\left(\frac{1}{x}\right)}{-\left(\frac{1}{x^2}\right)} = \lim\limits_{x\to 0^+} -\dfrac{x^2}{x} = \lim\limits_{x\to 0^+} -x = 0$

3. [8 points] Evaluate the following antiderivatives (aka indefinite integrals).

a. $\int (x^3 - 4e^x - \cos(x) + \ln(2))\, dx = \boxed{\dfrac{x^4}{4} - 4e^x - \sin(x) + x\ln(2) + C}$

b. $\int \dfrac{x^{3.6}+\sqrt{x}}{x^2}\, dx = \int (x^{1.6} + x^{-\frac{3}{2}})\, dx$

$= \dfrac{x^{2.6}}{2.6} - \dfrac{x^{-1/2}}{(1/2)} = \dfrac{x^{2.6}}{2.6} - 2x^{-1/2}$