Name: $\qquad$
Solve the following equations for $x$ or state that none exist.

1. $5 e^{x}-2=0, e^{x}=2 / 5, x=\ln (2 / 5)$
2. $5 e^{x}+4=0, e^{x}=-4 / 5$, no solution
3. $5 \ln (x)-6=0, \ln (x)=6 / 5, \quad x=e^{6 / 5}$
4. $5 \ln (x)+7=0, \quad \ln (x)=-7 / 5$

This page contains information and techniques you will need for Sections 4.5 and 4.6.

1. Write in your own words how to find the critical numbers of a function $f(x)$ and why they are important.
Look for $x$-values in the domain where $f^{\prime}(x)=0$ or $f^{\prime}(x)=D N E$. Critical numbers are where we look for local/absolute maximums or minimums.
2. Draw a graph of a function $f(x)$ with domain $(-\infty, \infty)$ such that
(i) $f^{\prime}(a)=f^{\prime}(b)=0$ and $f^{\prime}(c)$ is undefined,
and
(i) $f$ has a local minimum at $x=a$, a local maximum at $x=c$ and neither at $x=b$.

3. Draw a graph of a function $f(x)$ with domain $(-\infty, \infty)$ such that

(b) $f^{\prime}(x)<0$ and $f^{\prime \prime}(x)>0$.


4. For each function below, find (a) its domain and (b) all its critical points.

$$
\begin{aligned}
& \text { (a) } f(x)=x^{3}-2 x^{2} \quad D:(-\infty, \infty) \\
& f^{\prime}(x)=3 x^{2}-4 x=x(3 x-4)=0 \quad \text { Crit }{ }^{\prime} s: x=0, x=4 / 3 \\
& x=0 \text { or } x=4 / 3
\end{aligned}
$$

$$
\text { (b) } f(x)=x^{1 / 5} \quad D \cdot(-\infty, \infty)
$$

$$
f^{\prime}(x)=\frac{1}{5} x^{-4 / 5}=\frac{1}{5 x^{4 / 5}}
$$

$f^{\prime}(x)$ undefined at $x=0$
(c) $f(x)=\arctan (x) \quad D:(-\infty, \infty)$
$f^{\prime}(x)=\frac{1}{1+x^{2}} ; f^{\prime}$ is never
crit\#'s: none
zero or undefined.
(d) $f(x)=\frac{x^{2}}{x^{2}-4}$ (Note: $f^{\prime}(x)=\frac{-8 x}{\left(x^{2}-4\right)^{2}}$ )
$D:(-\infty,-2) \cup(-2,2) \cup(2, \infty)$
$f^{\prime}(x)=0$ when $x=0$.
crit\#'s: $x=0$
$f^{\prime}(x)$ is never undefined
in its domain
(e) $f(x)=e^{(1-x)^{2}} \quad \mathrm{D}:(-\infty, \infty)$
$\longrightarrow$ crit\#'s: $x=1$

$$
\begin{aligned}
f^{\prime}(x) & =e^{(1-x)^{2}} \cdot \frac{d}{d x}\left((1-x)^{2}\right) \\
& =e^{(1-x)^{2}}\left(2(1-x)^{\prime}(-1)\right)=-2(1-x) e^{(1-x)^{2}}=0 \text { when } x=1
\end{aligned}
$$

(f) $f(x)=\sqrt{x^{2}-4}=\left(x^{2}-4\right)^{1 / 2}$

$$
f^{\prime}(x)=\frac{1}{2}\left(x^{2}-4\right)^{-1 / 2}(2 x)=\frac{x}{\sqrt{x^{2}-4}}=0
$$

we need $x^{2}-4 \geq 0$

So $x^{2} \geqslant 4$ so

$$
\begin{aligned}
& x \geqslant 2 \text { or } x \leqslant-2 \\
& D:(-\infty,-2) \cup(2, \infty)
\end{aligned}
$$

when $x=0$.
But $x=0$ isn't in the domain of $f(x)$.
crit \#'s: none
5. For each derivative below, determine the intervals for which that derivative is positive and negative.
(a) $f^{\prime}(x)=x^{-4 / 5}$ is undefined at $x=0$

$f^{\prime}(x)>0$ for all $x$ in the domain
(b) $y^{\prime \prime}=\frac{8\left(3 x^{2}+4\right)}{\left(x^{2}-4\right)^{3}} \quad y^{\prime \prime}$ is undefined when $x= \pm 2$


$$
\begin{aligned}
& y^{\prime \prime}(-3)=\frac{t+t}{t}>0 \\
& y^{\prime \prime}(0)=\frac{t \cdot t}{-}<0 \\
& y^{\prime \prime}(3)=+t /+>0
\end{aligned}
$$

(c) $g^{\prime}(x)=3 x^{2} e^{2 x}+2 x^{3} e^{2 x}=x^{2} e^{2 x}(3+2 x)=0$

$$
x=0,-3 / 2
$$

$$
\begin{aligned}
& g^{\prime}(x)>0 \text { on }\left(\frac{-3}{2}, \infty\right) \text { and } \\
& g^{\prime}(x)<0 \text { on }\left(-\infty, \frac{-3}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& g^{\prime}(-10)=(t)(t)(-)<0 \\
& g^{\prime}(-1)=(t)(t)(t)>0 \\
& g^{\prime}(1)=(t)(t)(t)>0
\end{aligned}
$$

6. Write a formula for a function $f(x)$ such that $f(x)$ has asymptotes $x=1, x=4$ and $y=0$.

$$
f(x)=\frac{1}{(x-1)(x-4)}
$$

7. Give an example of a graph with two different horizontal asymptotes.

$$
f(x)=\arctan (x)
$$

$0^{2}$

8. Evaluate each limit below.
(a) $\lim _{x \rightarrow 2^{+}} \frac{5}{x-2}=+\infty$
(d) $\lim _{x \rightarrow \infty} \frac{5}{x-2}=0$
(b) $\lim _{x \rightarrow 2^{-}} \frac{5}{x-2}=-\infty$
(e) $\lim _{x \rightarrow-\infty} \frac{5}{x-2}=0$
(f) $\lim _{x \rightarrow \infty}\left(8+\frac{5}{x-2}\right)=\varnothing$
(c) $\lim _{x \rightarrow 2} \frac{5}{x-2}=$ DNE
(g) $\lim _{x \rightarrow \infty}\left(x+\frac{5}{x-2}\right)=\infty$

