1. **TYPE:** Secant lines and tangent lines. Let $f(x) = 1 + \frac{4}{x}$. $= 1 + 4x^{-1}$

   (a) Find the slope of the secant line between $P(1, f(1)$ and $Q = (2, f(2))$.

   (b) Write an equation of the tangent line to the graph of $f(x)$ at $x = 2$.

   (c) Sketch $f(x)$, the tangent line and the secant line on the same axes.

   (d) If f represented position and x represented time, which of the calculations above would be average velocity and which would be instantaneous velocity?

$f'(x) = -4x^{-2}$

$f(1) = 1 + \frac{4}{1} = 5$

$f(2) = 1 + \frac{4}{2} = 3$

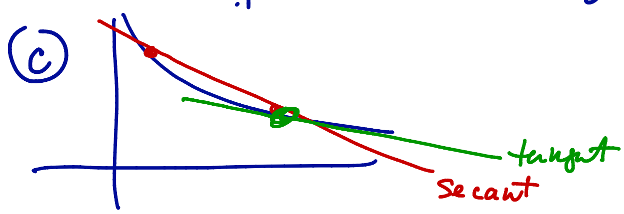
$f'(2) = \frac{-4}{2^2} = -1$

(a) $m_{sec} = \frac{5-3}{1-2} = -2$

(b) $m_{tan} = -1$, point $Q(2,3)$

line: $y - 3 = -1(x-2)$ or $y = 5 - x$

(c)



(d) (a) = average velocity

(b) = instantaneous velocity

2. **TYPE:** Definition of the derivative.

   (a) State the definition of the derivative.

   (b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{3x}$. No credit will be given for answers not using the definition. Points will be deducted for poorly written answers.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = \lim_{h \to 0} \frac{\sqrt{3x+3h} - \sqrt{3x}}{h} \cdot \frac{(\sqrt{3x+3h} + \sqrt{3x})}{(\sqrt{3x+3x} + \sqrt{3x})}$$

$$= \lim_{h \to 0} \frac{3x + 3h - 3x}{h(\sqrt{3x+3h} + \sqrt{3x})} = \lim_{h \to 0} \frac{3h}{h(\sqrt{3x+3h} + \sqrt{3x})} =$$

$$= \lim_{h \to 0} \frac{3}{\sqrt{3x+3h} + \sqrt{3x}} = \frac{3}{\sqrt{3x} + \sqrt{3x}} = \frac{3}{2\sqrt{3x}}$$

3. **TYPE:** Derivative as rate of change. The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

   (a) Suppose $f'(5) = 2000$. What are the units of the derivative?

   # bacteria/hours

   (b) In the context of the problem, explain what $f'(5) = 2000$ means using complete sentences.

   Five hours after the experiment started, the population of bacteria is increasing at a rate of 2000 bacteria/hr.

   (c) If $f(5) = 40,000$, how would you estimate $f(7)$ given the available information?

   $f(7) \approx 40000 + 2(2000) = 44{,}000$ bacteria.

4. **TYPE:** Evaluating limits. Evaluate the limits below. Justify your answer with words and/or algebra.

   (a) $\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to -3} \frac{x(x+3)}{(x+3)(x-4)} = \lim_{x\to -3} \frac{x}{x-4} = \frac{-3}{-7} = \frac{3}{7}$

   (b) $\lim_{x\to 1^+} \ln\left(\frac{5-x^2}{1+x}\right) = \ln\left(\frac{5-1}{1+1}\right) = \ln\left(\frac{4}{2}\right) = \ln(2)$

   (c) $\lim_{x\to 4^-} \frac{\sqrt{x}}{(x-4)^5} = -\infty$

   as $x \to 4^-$ (#'s like 3.9, 3.99...)

   $\sqrt{x} \to 4$ and $(x-4)^5 \to 0^-$

   (d) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5} = \lim_{x\to 5} \left(\frac{1}{x-5}\right)\left(\frac{1}{x}-\frac{1}{5}\right) = \lim_{x\to 5} \left(\frac{1}{x-5}\right)\left(\frac{5-x}{5x}\right) = \lim_{x\to 5} \frac{-(x-5)}{(x-5)(5x)} = \lim_{x\to 5} \frac{-1}{5x}$

   $= -\frac{1}{25}$

   (e) $\lim_{x\to 7} \left(x + \frac{x-7}{\sqrt{x}-\sqrt{7}}\right) = 7 + \lim_{x\to 7} \frac{(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})}{\sqrt{x}-\sqrt{7}}$

   $= 7 + \lim_{x\to 7} \sqrt{x}+\sqrt{7} = 7 + 2\sqrt{7}$

5. **TYPE:** Position, Velocity, Acceleration
A particle is moving back and forth along a straight line. The position function of a particle is given by $s(t) = \frac{1}{3}t^3 - 4t^2 + 12t$ where t is measured in seconds and s in meters.

(a) What is the velocity function of the particle?

$v(t) = s'(t) = t^2 - 8t + 12$

(b) What is the acceleration function of the particle?

$a(t) = s''(t) = 2t - 8$

$$\begin{array}{r} 12 \\ 9 \\ \hline 21 \\ -24 \end{array}$$

(c) At $t = 3$, is the particle speeding up or slowing down? (Justify!)

$v(3) = 3^2 - 8(3) + 12 = -3$

$a(3) = 2 \cdot 3 - 8 = -2$

Ans Speeding up.
Because velocity and acceleration have the same sign.

(d) When does the particle turn around?

Look for $v = 0$.

$0 = t^2 - 8t + 12 = (t-6)(t-2)$

So $t = 6s$ and $t = 2s$

(e) When is the particle moving to the right?

Look for $v > 0$

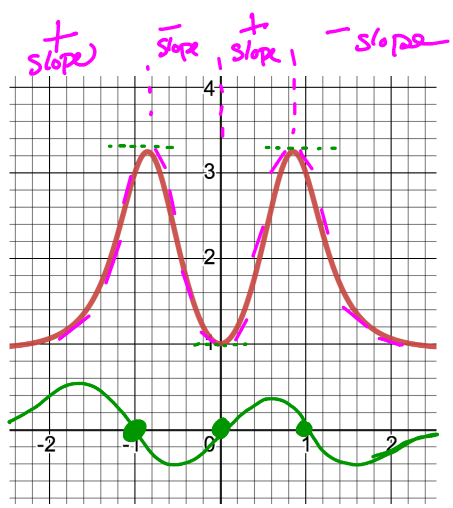
$+ + + \; 0 \; - - - \; 0 \; + + +$ ~ Sign of v

$\longleftarrow \; 2 \; \; 6 \; \longrightarrow$

Ans The particle is moving to the right when $t < 2$ or $t > 6$.

6. **TYPE:** Derivative as Function

Using the graph of $f(x)$ below, sketch the graph of $f'(x)$.



7. **TYPE:** Derivatives
Find the derivatives for each function below. You do not need to simplify but you must use parentheses correctly.

(a) $g(x) = \frac{2}{x} - 3\left(\frac{x^2+1}{5}\right) + 2\sqrt{2}$ $= 2x^{-1} - \frac{3}{5}x^2 - \frac{3}{5} + 2\sqrt{2}$

$g'(x) = -2x^{-2} - \frac{6}{5}x$

product rule!

(b) $h(x) = \cos(x) - \sqrt{x}\sin(x)$ $= \cos(x) - x^{\frac{1}{2}}\sin(x)$

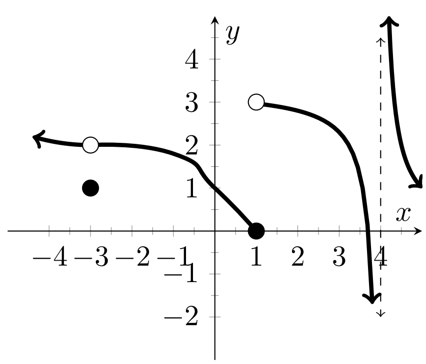
$h'(x) = -\sin(x) - \frac{1}{2}x^{-1/2}\sin(x) - x^{\frac{1}{2}}\cos(x)$

(c) $k(x) = x^2 - \frac{x^2+2}{5+\sin(x)}$

$k'(x) = 2x - \frac{(5+\sin(x))(2x) - (x^2+2)(\cos x)}{(5+\sin(x))^2}$

8. **TYPE:** Graphical Limits

For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to -3} f(x) =$ 2

(b) $f(-3) =$ 1

(c) $\lim_{x\to 1^-} f(x) =$ 0

(d) $\lim_{x\to 1^+} f(x) =$ 3

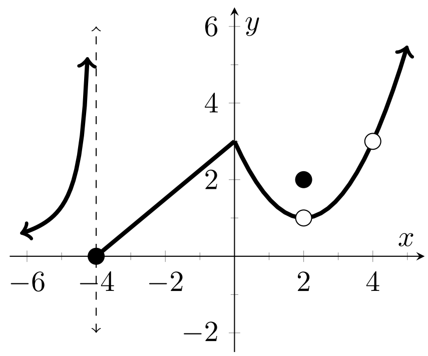
(e) $\lim_{x\to 1} f(x) =$ DNE

(f) $f(1) =$ 0

(g) $\lim_{x\to 4^-} f(x) =$ $-\infty$

(h) $\lim_{x\to 4^+} f(x) =$ $+\infty$

9. **TYPE:** Graphical Contintuity & Differentiability
A graph of the function $f(x)$ is displayed below.



(a) From the graph of f, state the numbers at which f is discontinuous and why.

$x=-4$ no limit

$x=2$ $\lim_{x \to 2} f(x) \neq f(2)$

$x=4$ $f(4)$ DNE

(b) From the graph of f, state the numbers at which f fails to be differentiable and why.

$x=-4, 2, 4$: no limit

$x=0$ corner

10. **TYPE:** One and Two Sided Limits
Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x \to 4} f(x)$ or explain why this limit does not exist.

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 3 = 3$

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{3(x-4)}{|x-4|} = -3$

as $x \to 4^-$ (#'s like 3.9, 3.99..)

The limit does not exist because the left- and right-hand limits are not the same.

11. **TYPE:** Intermediate Value Theorem
Using complete sentences, use the Intermediate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.

Let $f(x) = e^x - 3 + 2x$.
When $f(x)=0$, the equation has a solution.
Note that $f(x)$ is continuous.
$f(0) = e^0 - 3 + 2\cdot 0 = 1 - 3 = -2$ and $f(1) = e^1 - 3 + 2 = e - 1 > 0$.
Since $f(0) < 0$, $f(1) > 0$, and f is continuous, the I.V.Thm implies there is some x-value in $(0,1)$ where $f(x) = 0$.