RECITATION: WEEK 5

1. **TYPE:** Secant lines and tangent lines. Let $f(x) = 1 + \frac{4}{x}$.
   (a) Find the slope of the secant line between $P(1, f(1)$ and $Q = (2, f(2))$.
   (b) Write an equation of the tangent line to the graph of $f(x)$ at $x = 2$.
   (c) Sketch $f(x)$, the tangent line and the secant line on the same axes.
   (d) If f represented position and x represented time, which of the calculations above would be average velocity and which would be instantaneous velocity?

2. **TYPE:** Definition of the derivative.
   (a) State the definition of the derivative.
   (b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{3x}$. No credit will be given for answers not using the definition. Points will be deducted for poorly written answers.

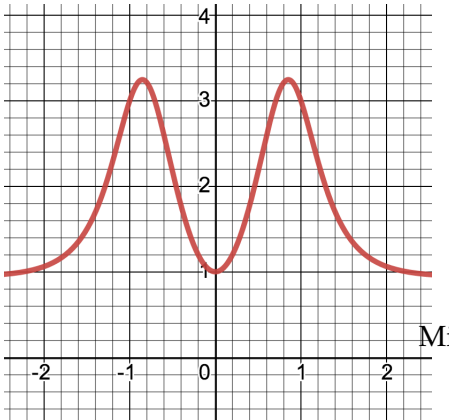
3. **TYPE:** Derivative as rate of change. The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.
   (a) Suppose $f'(5) = 2000$. What are the units of the derivative?
   (b) In the context of the problem, explain what $f'(5) = 2000$ means using complete sentences.
   (c) If $f(5) = 40,000$, how would you estimate $f(7)$ given the available information?

4. **TYPE:** Evaluating limits. Evaluate the limits below. Justify your answer with words and/or algebra.
   (a) $\lim_{x\to -3} \dfrac{x^2+3x}{x^2-x-12}$
   (b) $\lim_{x\to 1^+} \ln\left(\dfrac{5-x^2}{1+x}\right)$
   (c) $\lim_{x\to 4^-} \dfrac{\sqrt{x}}{(x-4)^5}$
   (d) $\lim_{x\to 5} \dfrac{\frac{1}{x}-\frac{1}{25}}{x-5}$
   (e) $\lim_{x\to 7} \left(x + \dfrac{x-7}{\sqrt{x}-\sqrt{7}}\right)$

5. **TYPE:** Position, Velocity, Acceleration
   A particle is moving back and forth along a straight line. The position function of a particle is given by $s(t) = \frac{1}{3}t^3 - 4t^2 + 12t$ where t is measured in seconds and s in meters.
   (a) What is the velocity function of the particle?
   (b) What is the acceleration function of the particle?
   (c) At $t = 3$, is the particle speeding up or slowing down?
   (d) When does the particle turn around?
   (e) When is the particle moving to the right?

6. **TYPE:** Derivative as Function

   Using the graph of $f(x)$ below, sketch the graph of $f'(x)$.



7. **TYPE:** Derivatives
Find the derivatives for each function below. You do not need to simplify but you must use parentheses correctly.

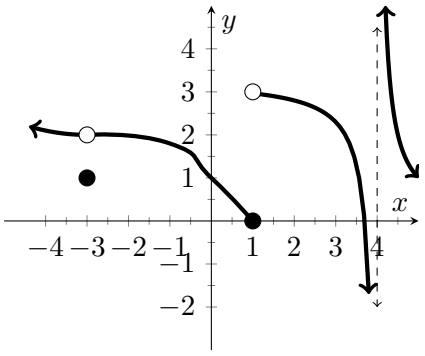
(a) $g(x) = \frac{2}{x} - 3\left(\frac{x^2+1}{5}\right) + 2\sqrt{2}$

(b) $h(x) = \cos(x) - \sqrt{x}\sin(x)$

(c) $k(x) = x^2 - \frac{x^2+2}{5+\sin(x)}$

8. **TYPE:** Graphical Limits

For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to -3} f(x) =$ _____

(b) $f(-3) =$ _____

(c) $\lim_{x\to 1^-} f(x) =$ _____

(d) $\lim_{x\to 1^+} f(x) =$ _____

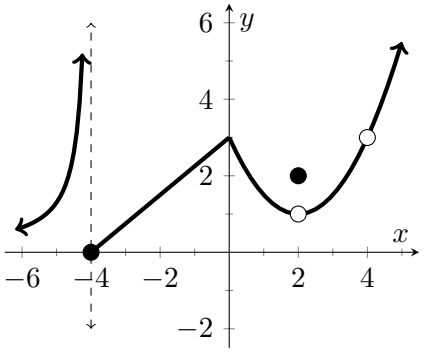
(e) $\lim_{x\to 1} f(x) =$ _____

(f) $f(1) =$ _____

(g) $\lim_{x\to 4^-} f(x) =$ _____

(h) $\lim_{x\to 4^+} f(x) =$ _____

9. **TYPE:** Graphical Contintuity & Differentiability
A graph of the function $f(x)$ is displayed below.



(a) From the graph of f, state the numbers at which f is discontinuous and why.

(b) From the graph of f, state the numbers at which f fails to be differentiable and why.

10. **TYPE:** One and Two Sided Limits
Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x\to 4} f(x)$ or explain why this limit does not exist.

11. **TYPE:** Intermediate Value Theorem
Using complete sentences, use the Intermediate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.