Math 252 Calculus 2 (Bueler)   Thursday, 6 October 2022

Midterm Exam 1

No book, notes, electronics, calculator, or internet access. 100 points possible. 70 minutes maximum.

1. (*7 pts*)   Compute the area between the curves $y = x^2$ and $y = \sqrt{x}$ on the interval $0 \le x \le 1$. (*Hint. Be careful about which curve is above the other.*)
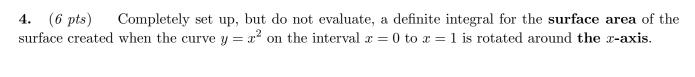
2. (*6 pts*)   Completely set up, but do not evaluate, a definite integral for the **length** of the curve $y = \sqrt{x}$ on the interval $x = 1$ to $x = 4$.

3. **(a)** (*4 pts*)   Sketch the region bounded by the curves $y = e^x$, $x = 0$, and $y = e$.   (*Hint. Double-check this part!*)

(b) (*4 pts*)   Use the **slicing (disks/washers)** method to completely set up, but not evaluate, a definite integral for the volume of the solid of revolution formed by rotating the region in part **(a)** around **the y-axis**.

(c) (*4 pts*)   Use the **shells** method to completely set up, but not evaluate, a definite integral for the volume of the same solid of revolution as in part **(b)**.

(d) (*4 pts*)   Evaluate one of the integrals in parts **(b)** or **(c)** to find the volume.

4. (*6 pts*)   Completely set up, but do not evaluate, a definite integral for the **surface area** of the surface created when the curve $y = x^2$ on the interval $x = 0$ to $x = 1$ is rotated around **the x-axis**.

5. It takes a force of 4 Newtons to hold a spring 3 centimeters from its equilibrium.

(a) (*3 pts*)   What is the spring constant k in Hooke's Law (i.e. $F = kx$)?

(b) (*6 pts*)   How much **work** is done to compress the spring 6 centimeters from its equilibrium? Simplify your answer and include units.

6. Evaluate and simplify the following indefinite and definite integrals.

(a) (*6 pts*) $\displaystyle\int_0^2 5^x \, dx =$

(b) (*6 pts*) $\displaystyle\int \cot\theta \, d\theta =$

(c) (*6 pts*) $\displaystyle\int \cos(7t)\sin(7t) \, dt =$

(d) (*6 pts*) $\displaystyle\int_0^{\pi/2} \sin^3 x \, dx =$

(e) (*6 pts*) $\displaystyle\int x^2 \sin x \, dx =$

(f) (*6 pts*) $\displaystyle \int \sec x \, dx =$

(g) (*6 pts*) $\displaystyle \int \sin(7x) \cos(3x) \, dx =$

7. (*8 pts*)   Evaluate and simplify the indefinite integral:

$$\int \frac{x^2 + x + 1}{x^3 + x}\, dx =$$

8. (*8 pts*)   Evaluate and *fully* simplify the indefinite integral.
(*Hint.* $(\tan \theta)' = \sec^2 \theta$ and $(\cot \theta)' = -\csc^2 \theta$.)

$$\int \frac{1}{x^2 \sqrt{1 - x^2}}\, dx =$$

Extra Credit. (*3 pts*)   Compute and simplify the integral

$$\int \sec^3 \theta \, d\theta =$$

You may find the following **trigonometric formulas** useful. However, there are other trig. formulas, not listed here, which you should have in memory, or which you know how to derive from these.

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(ax) \sin(bx) = \frac{1}{2} \cos((a - b)x) - \frac{1}{2} \cos((a + b)x)$$

$$\sin(ax) \cos(bx) = \frac{1}{2} \sin((a - b)x) + \frac{1}{2} \sin((a + b)x)$$

$$\cos(ax) \cos(bx) = \frac{1}{2} \cos((a - b)x) + \frac{1}{2} \cos((a + b)x)$$