Name: ____

Math 252 Calculus 2 (Bueler) 17 November 2022

Midterm Exam 2

No book, electronics, calculator, or internet access. Only "Summary of Convergence Tests" notes allowed. 100 points possible. 70 minutes.

1. (*5 pts*) Verify that $y = e^{2x^2}$ is a solution to the differential equation $y' - 4xy = 0$.

2. Compute and simplify the improper integrals, or show they diverge. Use correct limit notation.

(a) (*5 pts*) $\displaystyle\int_0^1 \frac{dx}{x^3} =$

(b) (*5 pts*) $\displaystyle\int_1^\infty 2xe^{-x^2}\,dx =$

3. Do the following series converge or diverge? Show your work, including naming any test you use.

(a) *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2}$

(b) *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{5n-1}$

(c) *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{2n}}$

(d) (*5 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n n!}{(n+2)!}$

(e) (*5 pts*) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n+3}{2n-1}\right)^n$

4. (*5 pts*) Does the following series converge or diverge? Show your work, including naming any test you use. (*Hint.* Previous problem? Or another test?)

$$\sum_{n=1}^{\infty} \frac{2n}{e^{(n^2)}}$$

5. (*5 pts*) Compute and simplify the value of the infinite series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^{n+1}$.

6. Consider the infinite series $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \ldots$

(a) (*5 pts*) Write the series using sigma ($\sum$) notation.

(b) (*5 pts*) Compute and simplify S_3, the partial sum of the first three terms.

(c) (*5 pts*) Does this series converge absolutely, conditionally, or neither (diverge)? Show your work, identify any test(s) used, and circle one answer.

CONVERGES ABSOLUTELY   CONVERGES CONDITIONALLY   DIVERGES

7. Use the well known geometric series $\dfrac{1}{1-r} = \displaystyle\sum_{n=0}^{\infty} r^n$ to find power series representations for the following functions. Show your work.

(a) *(5 pts)* $\dfrac{1}{1+x^3}$

$\dfrac{1}{1+x^3} =$

(b) *(7 pts)* $\ln(1+x)$

$\ln(1+x) =$

8. *(7 pts)* If $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$, find a power series representation for $f'(5x)$

$f'(5x) =$

9. Find the interval of convergence of the following power series.

(a) (*8 pts*) $$\sum_{n=1}^{\infty} \frac{2^n x^n}{n!}$$

(b) (*8 pts*) $$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\,3^n}$$

Extra Credit. (*3 pts*) The function $f(x) = \arctan(x)$ can be represented by the power series

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}.$$

Suppose I choose $x = 1/\sqrt{3}$ and compute the partial sum $S_{20} = \displaystyle\sum_{n=0}^{20} \frac{(-1)^n (1/\sqrt{3})^{2n+1}}{2n+1}$ as an approximation to $\arctan(1/\sqrt{3}) = \dfrac{\pi}{6}$.

How accurate is this approximation? Use a known fact about remainders of alternating series.

$$\left| S_{20} - \frac{\pi}{6} \right| \leq \boxed{}$$

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