

Name: _____

Rules:

You have 2 hours to complete this midterm.

Partial credit will be awarded, but you must show your work.

You may have a single sheet of paper written on the front only.

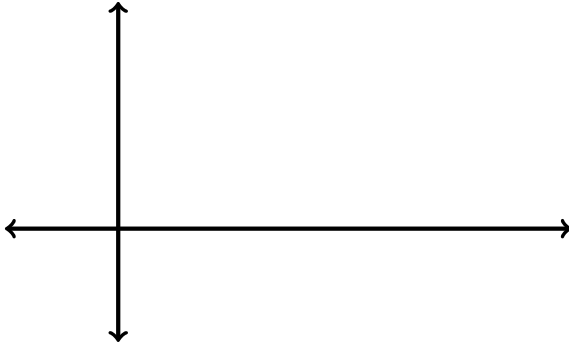
Calculators and books are not allowed.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	5	
3	5	
4	10	
5	10	
6	10	
7	8	
8	8	
9	6	
10	12	
11	8	
12	8	
Extra Credit	5	
Total	100	

1. (10 points) Let R be the region of the plane bounded by $y = 3\sqrt{x}$, the y -axis, and $y = 3$.
- (a) Sketch the region R . Label at least three points on your graph.



- (b) Find the area of the region R . Your final answer should be simplified.
- (c) Find the volume of the solid obtained by rotating the region R about the x -axis. Your final answer should be simplified.

2. (5 points) A 1-meter long rod oriented along the x -axis on the interval $[0, 1]$ has density $\rho(x) = xe^{3x}$ grams per meter at position x meters. Find the mass of the rod. Include units in your answer.

3. (5 points) Evaluate the definite integral: $\int_0^1 \frac{5x + 1}{(x + 1)(2x + 1)} dx$

4. (10 points) Evaluate the following indefinite integrals.

(a) $\int \sec^4(\theta) d\theta$

(b) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

5. (10 points) Determine whether the series is convergent or divergent. Note that to earn full credit, your work must include the name of the test being applied, a clear application of the test, and a conclusion.

(a)
$$\sum_{n=0}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

(b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2n + 1}}$$

6. (10 points) Determine whether the series is convergent or divergent. Note that to earn full credit, your work must include the name of the test being applied, a clear application of the test, and a conclusion.

(a)
$$\sum_{n=0}^{\infty} \ln\left(\frac{2n}{3n+5}\right)$$

(b)
$$1 + e + \frac{e^2}{2!} + \frac{e^3}{3!} + \frac{e^4}{4!} + \cdots$$

7. (8 points) Evaluate the improper integral $\int_2^{\infty} \frac{dx}{x(\ln(x))^2}$ or demonstrate that it is divergent. Use correct limit notation.

8. (8 points) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$

9. (6 points) Find the Taylor series for $f(x) = e^{5x}$ centered at $a = -1$.

10. (12 points) Answer the questions about the parametric equations $x(t) = t^2$ and $y(t) = t^3 + t$, where $0 \leq t$.

(a) Make a rough sketch of the curve defined by the parametric equations. Plot at least 4 points and indicate with arrows the direction in which the curve is traced as t increases.

(b) Write an equation of the line tangent to the curve when $t = 1$.

(c) Use the second derivative to demonstrate that the curve is concave up when $t = 1$.

11. (8 points) Make a careful and reasonably large sketch of the polar curve $r = 1 + \cos(\theta)$. To earn full credit, you must label at least 4 points on your graph.

12. (8 points) Find the area inside the polar curve in the previous problem.

Extra Credit (5 points) Determine all the values of p for which the series $\sum_{n=0}^{\infty} \frac{4^{pn}}{3^n}$ converges or explain why it is not possible for the series to converge for any value of p .