Math F252 Midterm I Fall 2023

Name: ____________________

Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten 3×5 notecard.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	20	
4	12	
5	8	
6	30	
7	10	
Extra Credit	5	
Total	100	

1. (10 points) Find the area between the curves $y = \dfrac{10}{x+1}$ and $y = 10 - 2x$ on the interval $[0, 4]$. Your final answer should be a reasonably simplified number.

2. (10 points) Find the surface area of the volume generated when the curve $y = \frac{x^3}{3}$ from $x = 0$ to $x = 2$ is revolved around the x-axis. Your final answer should be a reasonably simplified number.

3. (20 points) Let R be the region in the first quadrant bounded by $y = \sqrt{x}$, $y = 0$, and $x = 16$.

   (a) Sketch the region R on the given graph.

   (b) For each problem below, set up – but do not evaluate – a definite integral for the volume of the solid of revolution formed by rotating the region R about the given axis using the given method. **Your final answer should be in a form that is immediately integrable without any additional algebra.**

      i. Axis of rotation: x-axis, Method: disks/washers.

      ii. Axis of rotation: x-axis, Method: shells.

      iii. Axis of rotation: y-axis, Method: disks/washers.

      iv. Axis of rotation: y-axis, Method: shells.

4. (12 points) A spring has a natural length of 30 cm (or 0.3 m). It takes a force of 40 N to hold the spring at a length of 40 cm (or 0.4 m).

   (a) What is the spring constant k in Hooke's Law?

   (b) How much **work** is done to stretch the spring to a length of 50 cm (or 0.5 m)? Simplify your answer and include units.

5. (8 points) Set up but do not evaluate the three integrals needed to compute the center of mass, $(\overline{x}, \overline{y})$, of the region R bounded by $y = 0$, $x = 0$, $x = 2$, and $f(x) = 2e^x$ with constant density $\rho = 1$. Then, fill in the blanks at the bottom to show how to compute the values of $(\overline{x}, \overline{y})$.

$m =$

$M_y =$

$M_x =$

$\overline{x} = \dfrac{\boxed{}}{\boxed{}}$ $\overline{y} = \dfrac{\boxed{}}{\boxed{}}$

6. (30 points) Evaluate the following indefinite integrals. Show your work and simplify your answers.

   (a) $\int x^2 e^x \, dx$

   (b) $\int \sin^3(\theta) \cos^2(\theta) \, d\theta$

(c) $\displaystyle\int \frac{4 - x^2}{x^3 + 2x}\,dx$

(d) $\displaystyle\int x\sec^2(x)\,dx$

7. (10 points) Evaluate the integral $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ using trigonometric substitution. You must **fully** simplify your answer.

Extra Credit (5 points)

(a) Use integration by parts to prove the reduction formula $\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$.

(b) Use the reduction formula to evaluate $\int (\ln x)^2 \, dx$.

$$\sin^2(x) = \tfrac{1}{2}(1 - \cos(2x)) \qquad \sin(ax)\cos(bx) = \tfrac{1}{2}(\sin((a-b)x) + \sin((a+b)x))$$

$$\cos^2(x) = \tfrac{1}{2}(1 + \cos(2x)) \qquad \sin(ax)\sin(bx) = \tfrac{1}{2}(\cos((a-b)x) - \cos((a+b)x))$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(ax)\cos(bx) = \tfrac{1}{2}(\cos((a-b)x) + \cos((a+b)x))$$