Math F252X-901 - Fall 2024
Exam 1

- No outside materials (e.g. books, notes, calculators, other electronic devices).
- SHOW ALL WORK. Credit may not be given for answers without sufficient work.
- Illegible work will not be graded.

Print Name:_____

Page	Points	Score
1	14	
2	16	
3	16	
4	10	
5	18	
6	19	
7	7	
Total:	100	

(8 pts) 1. Find the area of the region in the first quadrant bounded by the curves $y = x^2 + 2x$ and x^3.

(6 pts) 2. Completely set up, but do not evaluate, a definite integral which gives the **arc length** of the curve $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1$.

(16 pts) 3. Let R be the region bounded by the curves $y = \sqrt{x}$, $x = 0$, and $y = 2$.

(a) Sketch the region R. (*Hint. Double-check this part before proceeding to part (b).*)

(b) Use the **slicing (disks/washers)** method to completely set up, but not evaluate, a definite integral for the volume of the solid of revolution formed by rotating R around **the x-axis**.

(c) Use the **shell** method to completely set up, but not evaluate, a definite integral for the volume of the same solid of revolution as in part **(b)**.

(d) **Evaluate one** of the integrals in parts **(b)** or **(c)** to find the exact volume.

(6 pts) 4. Completely set up, but do not evaluate, a definite integral for the **surface area** of the surface created when the curve $y = \sin(x)$ on the interval $x = 0$ to $x = \pi$ is rotated around **the x-axis**.

(10 pts) 5. It takes a force of 4 Newtons to hold a spring 3 centimeters from its equilibrium.

(a) What is the spring constant k in Hooke's Law (i.e. $F = kx$)?

(b) How much **work** is done to compress the spring 6 centimeters from its equilibrium? Simplify your answer and include units.

(10 pts) 6. Compute the center of mass $(\bar{x}, \bar{y})$ for the region in the first quadrant bounded by $y = 0$, $x = 1$ and $y = 2x^2$.

7. Compute the following integrals and antiderivatives.

(6 pts) (a) $\int (2^x + \frac{5}{x} + e^{-3})\, dx$

(6 pts) (b) $\int_0^{\pi/2} x \sin(x)\, dx$

(6 pts) (c) $\int x^2 e^{(3x)}\, dx$

(6 pts) (d) $\int \sin^2(x)\cos^5(x)\,dx$

(6 pts) (e) $\int \cos(6x)\sin(2x)\,dx$

(7 pts) (f) $\int \frac{dx}{x^2\sqrt{1-x^2}}$

(7 pts) (g) $\displaystyle\int \frac{x}{x^2 - x - 6}\, dx$

BONUS (5 points): $\displaystyle\int \frac{x^2}{4 + 9x^2}\, dx$

You may find the following trigonometric formulas useful.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$