

Math F252X-901 - Fall 2024

Exam 2

- No outside materials (e.g. books, notes, calculators, other electronic devices).
- SHOW ALL WORK. Credit may not be given for answers without sufficient work. Cite any tests/theorems you use.
- Illegible work will not be graded.

Print Name: _____

Page	Points	Score
1	18	
2	16	
3	12	
4	12	
5	12	
6	12	
7	12	
8	6	
Total:	100	

(6 pts) 1. Evaluate the integral $\int_2^{\infty} \frac{1}{x(\ln(x))^2} dx$.

(6 pts) 2. Evaluate the integral $\int_0^3 \frac{1}{x-1} dx$.

(6 pts) 3. Find a closed form (explicit formula) for the sequence given by the recurrence relation $a_1 = 6000$,
 $a_{n+1} = \frac{a_n}{10}$.

(4 pts) 4. Find the first three partial sums for the series $\sum_{k=1}^{\infty} (-1)^k (2^{k-1})$.

$$S_1 = \underline{\hspace{2cm}}$$

$$S_2 = \underline{\hspace{2cm}}$$

$$S_3 = \underline{\hspace{2cm}}$$

5. Determine whether the following series converge or diverge. If a series converges, find its sum/limit.

(6 pts) (a) $\sum_{n=1}^{\infty} 3^{1/n} - 3^{1/(n+1)}$

(6 pts) (b) $\sum_{n=1}^{\infty} \frac{5}{2^n}$

6. Determine whether the following series converge or diverge.

(6 pts) (a) $\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$

(6 pts) (b) $\sum_{k=5}^{\infty} \frac{2k}{k^2 - 4k}$

(6 pts) (c) $\sum_{k=50}^{\infty} \frac{1}{e^{1/k}}$

(6 pts) (d) $\sum_{k=1}^{\infty} \frac{k}{2^k}$

(6 pts) (e) $\sum_{k=0}^{\infty} \frac{5^k}{k!}$

7. Determine whether the following series converge conditionally, converge absolutely, or diverge.

(6 pts) (a) $\sum_{k=0}^{\infty} \frac{(-5)^k}{k!}$

(6 pts) (b) $\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$

(6 pts) 8. Find the center, interval, and radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$.

center: _____

radius: _____

interval: _____

9. Find power series representations for the following functions. State the radius of convergence for each. (Recall that $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$.)

(6 pts) (a) $f(x) = \frac{1}{1+2x}$

(6 pts) (b) $g(x) = \frac{2}{(1+2x)^2}$.

(6 pts) 10. Find the Maclaurin series (Taylor series centered at 0) for the function $f(x) = \sin(x)$.

BONUS: (6 points) Determine whether the series $\sum_{k=1}^{\infty} \log\left(\frac{n}{n+1}\right)$ converges or diverges.
