Math F252 # Midterm II **Spring 2024**

Name: ______________________________

Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question, or use the box provided.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	4	
3	18	
4	18	
5	13	
6	12	
7	5	
8	6	
9	12	
Extra Credit	*3*	
Total	100	

1. Compute and simplify the improper integrals, or show they diverge. Use correct limit notation.

(a) (*6 pts*) $\displaystyle\int_0^1 \frac{dx}{x^{1/3}} =$

(b) (*6 pts*) $\displaystyle\int_1^\infty \frac{x\,dx}{1+x^2} =$

2. (*4 pts*) Find a formula for the general term a_n of the sequence

$\{0, 3, 8, 15, 24, 35, 48, \dots\}$

3. Do the following series converge or diverge? Show your work, including naming any test you use.

(a) (*6 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2}$

(b) (*6 pts*) $\displaystyle\sum_{n=1}^{\infty} \ln(n)$

(c) (*6 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

4. Do the following series converge or diverge? Show your work, including naming any test you use.

(a) (*6 pts*) $$\sum_{n=0}^{\infty} \frac{2^n}{(n+2)!}$$

(b) (*6 pts*) $$\sum_{n=0}^{\infty} \left(\frac{n+1}{2n+3}\right)^n$$

(c) (*6 pts*) $$\sum_{n=1}^{\infty} \frac{n}{e^{(n^2)}}$$

5. Consider the infinite series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$

(a) (*4 pts*) Write the series using sigma ($\sum$) notation.

(b) (*4 pts*) Compute and simplify S_3, the partial sum of the first three terms.

(c) (*5 pts*) Does the series converge absolutely, conditionally, or neither (diverge)? Show your work, identify any test(s) used, and circle one answer.

CONVERGES ABSOLUTELY CONVERGES CONDITIONALLY DIVERGES

6. Use the well known geometric series $\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n$ to find power series representations for the following functions. Show your work. (*Hint on part* **(b)**: *Use the answer from part* **(a)**.)

(a) (*6 pts*) $\frac{1}{1+x^2}$

$$\frac{1}{1+x^2} =$$

(b) (*6 pts*) $\arctan x$

$$\arctan x =$$

7. (*5 pts*) Compute and simplify the value of the infinite series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^{n+1}$.

8. (*6 pts*) If $f(x) = \displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$, find a *simplified* power series representation for $f'(-x^2)$.

$f'(-x^2) =$

9. Find the **radius** and **interval** of convergence of the following power series.

(a) (*6 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$

R =	interval:

(b) (*6 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n\,2^n}$

R =	interval:

Extra Credit. (*3 pts*) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges to $\pi/4$. Suppose you wanted to use this series to obtain an estimate of $\pi/4$ that is within 0.0001 of the actual value. Determine the fewest number of terms you would need to sum in order to obtain this level of accuracy. Explain your reasoning.

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