opring 2024

. (

1. Compute and simplify the improper integrals, or show they diverge. Use correct limit notation.

(a) (6 pts)
$$\int_{0}^{1} \frac{dx}{x^{1/3}} = \lim_{\substack{a \to o^{+} \\ a \to o^{+} \\ a \to o^{+} \\ \hline a$$

2. (4 *pts*) Find a formula for the general term a_n of the sequence

 $\{0, 3, 8, 15, 24, 35, 48, \dots\}$ 0 = |-|3=4-1=22-1 8=9-1=32-1 $|5 = |6 - 1 = 4^2 - 1$ ۷

an=

an

=(n-1)(n+1)

0

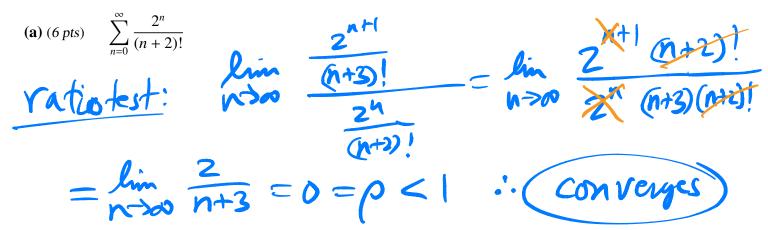
Con vorges

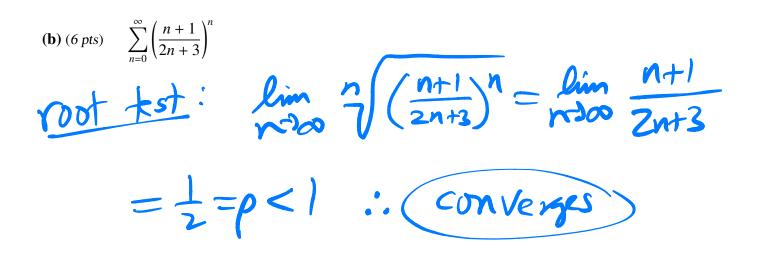
3. Do the following series converge or diverge? Show your work, including naming any test you use.

(**a**) (6 pts) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2}$ limit compare to $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} (p=3/2)$ $\frac{\sqrt{n+1}}{n^2} = \lim_{n \to \infty} \frac{\sqrt{n+1}}{\sqrt{n}} \frac{n}{\sqrt{n}} = \int_{n \to \infty} \frac{n+1}{\sqrt{n}} = \int_{n \to \infty} \frac{1}{\sqrt{n}} \frac{1}{\sqrt$ **(b)** (6 pts) $\sum_{i=1}^{\infty} \ln(n)$ lim hn = to : divers test (c) (6 pts) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ $bn = \frac{1}{\sqrt{n+1}} \ge 0$, $\lim_{n \to \infty} bn^{-0}$, bn decreases

by alternating series

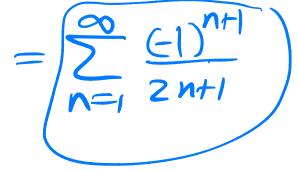
4. Do the following series converge or diverge? Show your work, including naming any test you use.





(c) (6 pts) $\sum_{n=1}^{\infty} \frac{n}{e^{(n^2)}}$ integral test: $\int_{1}^{\infty} x e^{-x^2} dx = \lim_{t \to \infty} \int_{1}^{t} e^{-y} \frac{dy}{z}$ $\int_{1}^{(u=x^2)} \frac{du}{z} = x dx$ (c) (6 pts) $\sum_{n=1}^{\infty} \frac{n}{e^{(n^2)}}$ $= \lim_{t \to \infty} \frac{(dn - xdx)}{2} = \lim_{t \to \infty} \frac{(dn - xdx)}{2} = \frac{t}{2e} < \infty$ also works: root, limit comparison, ratio

- 5. Consider the infinite series $1 \frac{1}{3} + \frac{1}{5} \frac{1}{7} + \frac{1}{9} \frac{1}{11} + \dots$
- (a) (4 *pts*) Write the series using sigma (Σ) notation.



(b) (4 pts) Compute and simplify S_3 , the partial sum of the first three terms.

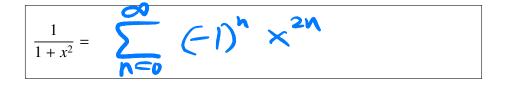


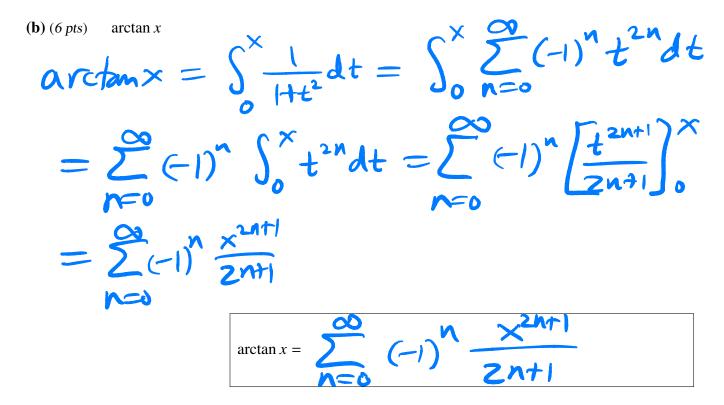
(c) (5 *pts*) Does the series converge absolutely, conditionally, or neither (diverge)? Show your work, identify any test(s) used, and circle one answer.

limit compare to b, decre series converges take abs. vals: Egoo: abs. senis duines CONVERGES CONVERGES DIVERGES CONDITIONALLY ABSOLUTELY

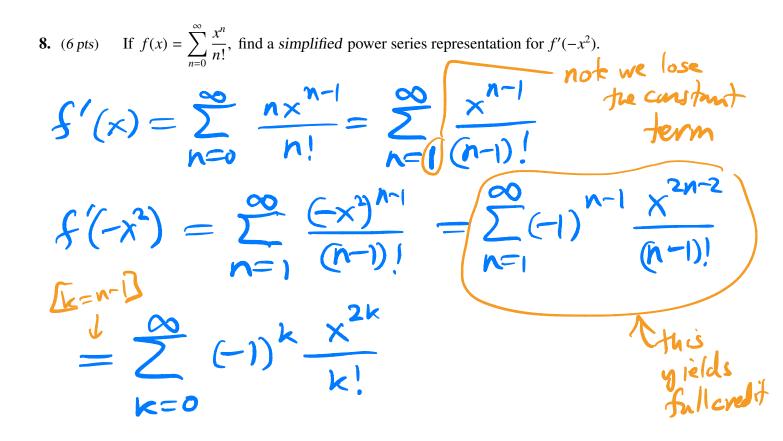
6. Use the well known geometric series $\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n$ to find power series representations for the following functions. Show your work. (*Hint on part* (b): Use the answer from part (a).)

(a) (6 pts)
$$\frac{1}{1+x^2}$$
 use $r = -x^2$
 $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$





7. (5 pts) Compute and simplify the value of the infinite series $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^{n+1} = \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \cdots$ Geometric with $a = \left(\frac{1}{5}\right)^2$, $r = \frac{1}{5}$: $Z \cdots = \frac{a}{1-r} = \frac{(15)^2}{1-15} = \frac{1}{5^2} \cdot \frac{5}{4} = \frac{1}{20}$

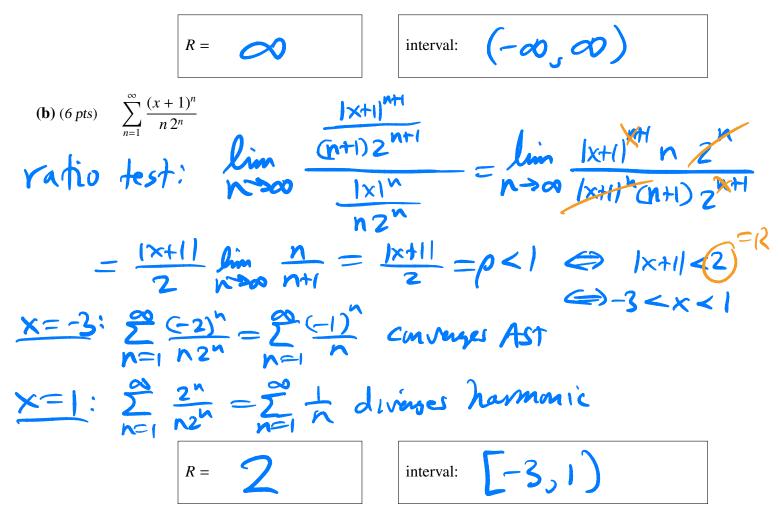


$$f'(-x^2) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{k!}$$

9. Find the radius and interval of convergence of the following power series.

(a) (6 pts)
$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$$
ratio test:
$$\lim_{n \to \infty} \frac{3^{n+1} |x|^{n+1}}{3^n |x|^n} = \lim_{n \to \infty} \frac{3^{n+1} |x|^{n+1}}{3^n |x|^n}$$

$$= \lim_{n \to \infty} \frac{3|x|}{n+1} = 0 = p < 1 \quad a |ways (for all x)$$



Extra Credit. (3 *pts*) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges to $\pi/4$. Suppose you wanted to use this series to obtain an estimate of $\pi/4$ that is within 0.0001 of the actual value. Determine the fewest number of terms you would need to sum in order to obtain this level of accuracy. Explain your reasoning.

mainidea RN = S - SNIRNI 2 bort, 3 for alternating series IRN 2(1+)+1 2N+3>10 N >8.5

BLANK SPACE