Math 252: Quiz 2 **8 September, 2022**

Name: SOLUTIONS

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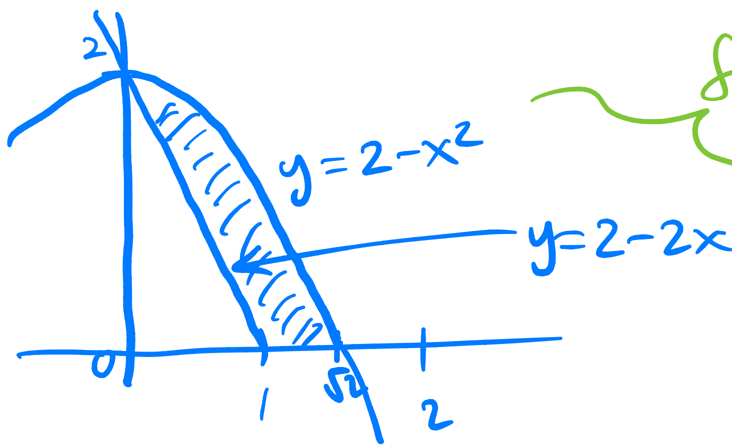
30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [7 points] Find the area of the region in the first quadrant enclosed by $y = 2 - 2x$, $y = 2 - x^2$, and the x-axis. (*Hint: Careful sketch first. Integrating with respect to either x or y will work.*)



for 2nd version:

$y=2-x^2 \Leftrightarrow x=\sqrt{2-y}$

$y=2-2x \Leftrightarrow x=1-\frac{y}{2}$

$$A = \int_0^1 (2-x^2)-(2-2x)\,dx + \int_1^{\sqrt{2}} 2-x^2\,dx$$

$$= \int_0^1 -x^2+2x\,dx + \int_1^{\sqrt{2}} 2-x^2\,dx$$

$$= \left[-\frac{x^3}{3}+x^2\right]_0^1 + \left[2x-\frac{x^3}{3}\right]_1^{\sqrt{2}}$$

$$= -\frac{1}{3}+1+\left(2\sqrt{2}-\frac{(\sqrt{2})^3}{3}\right)-\left(2-\frac{1}{3}\right)$$

$$= \frac{2}{3}+2\sqrt{2}-\frac{2\sqrt{2}}{3}-2+\frac{1}{3} = \frac{2}{3}(2\sqrt{2})-1 = \frac{4\sqrt{2}}{3}-1$$

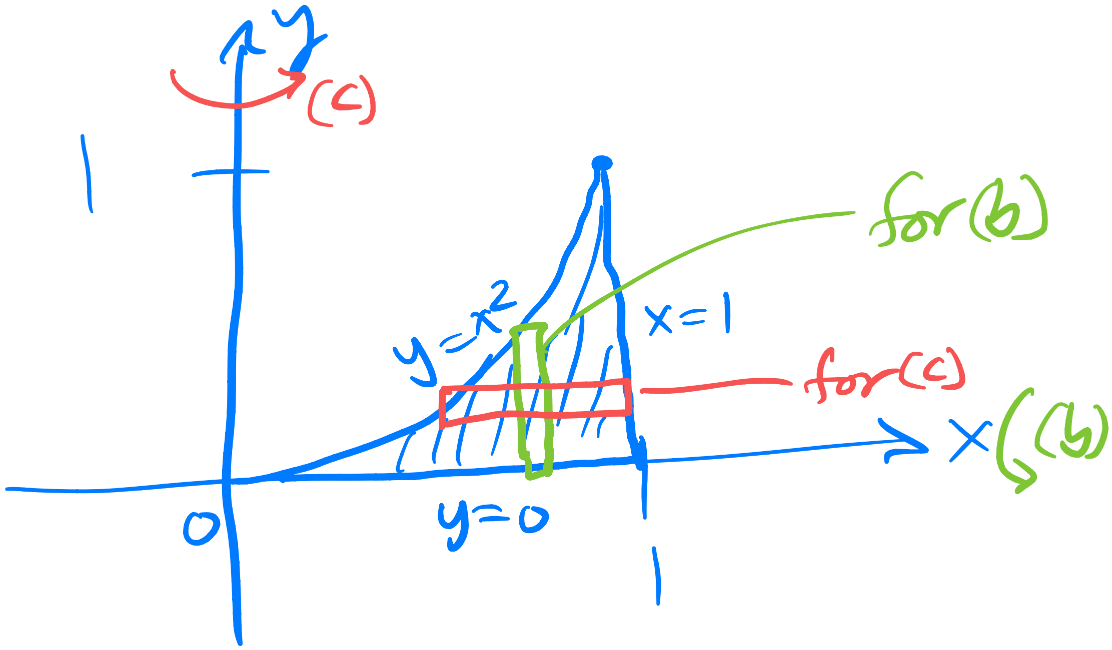
or:

$$A = \int_0^2 \sqrt{2-y}-\left(1-\frac{y}{2}\right)dy = \int_2^0 \sqrt{u}\,(-du) - \int_0^2 1-\frac{y}{2}\,dy$$

$$= \int_0^2 \sqrt{u}\,du + \int_0^2 \frac{y}{2}-1\,dy = \left[\frac{2}{3}u^{3/2}\right]_0^2 + \left[\frac{y^2}{4}-y\right]_0^2 = \frac{2}{3}2\sqrt{2}+1-2 = \frac{4\sqrt{2}}{3}-1$$

2. [13 points]

a. Sketch the region bounded by $y = x^2$, $y = 0$, and $x = 1$.



b. Find the volume of the solid formed by revolving the region in part **a.** around the x-axis. (*Hint: Use discs or washers.*)

$$V = \int_0^1 \pi (x^2)^2\,dx = \pi \int_0^1 x^4\,dx$$

$$= \pi \left[\frac{x^5}{5}\right]_0^1 = \frac{\pi}{5}$$

c. Find the volume of the solid formed by revolving the region in part **a.** around the y-axis. (*Hint: Use discs or washers.*)

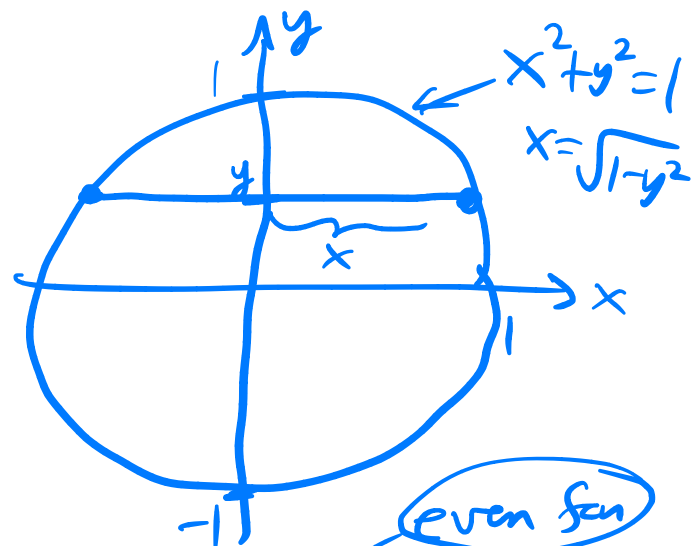
solve $y=x^2$ for x

$$V = \int_0^1 \pi\left(1^2 - (\sqrt{y})^2\right) dy$$

$$= \pi \int_0^1 1 - y \, dy = \pi \left[y - \frac{y^2}{2}\right]_0^1$$

$$= \pi\left[1 - \frac{1}{2}\right] = \frac{\pi}{2}$$

3. [5 points] A solid has a base which is the unit circle in the x, y plane, and each cross-section parallel to the x-axis is a square. Find the volume.



$$A(y) = (2x)^2 = \left(2\sqrt{1-y^2}\right)^2$$

area of square at slice at y

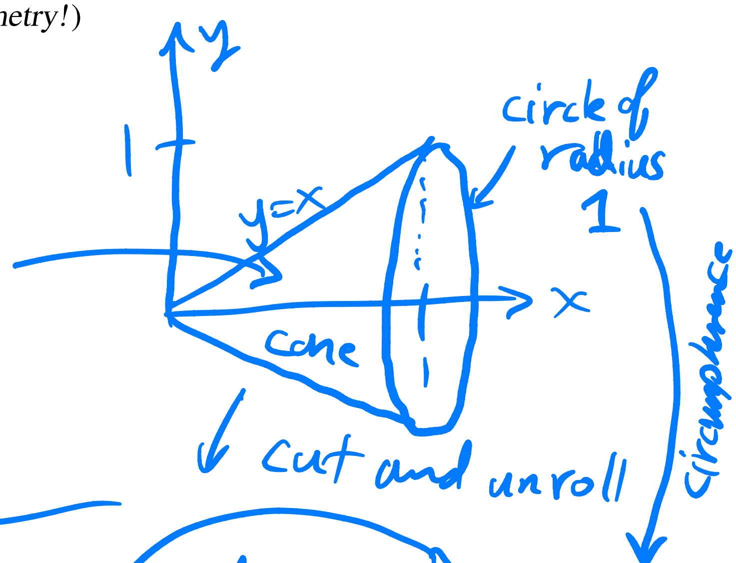
$$= 4(1-y^2)$$

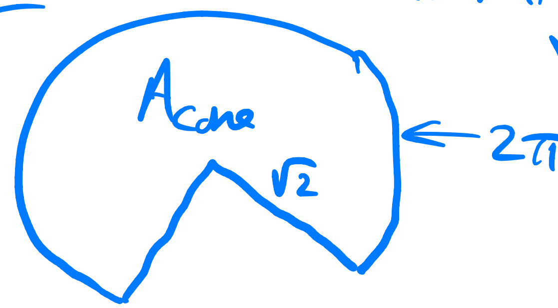
even fcn

$$V = \int_{-1}^1 A(y)\, dy = 4\int_{-1}^1 1-y^2 \, dy = 8\int_0^1 1-y^2 \, dy$$

$$= 8\left[y - \frac{y^3}{3}\right]_0^1 = 8\left[1 - \frac{1}{3}\right] = \frac{16}{3}$$

EC. [1 points] (**Extra Credit**) Rotating the line $y = x$, on the interval $0 \leq x \leq 1$, around the x-axis generates a cone. Find the area of this cone; do not include the area of the "base" of the cone at $x = 1$. (*Hint: No need to integrate! Unroll and do geometry!*)





$$A_{cone} = \frac{2\pi}{2\pi\sqrt{2}} \cdot \pi(\sqrt{2})^2$$

$$= \sqrt{2}\,\pi$$

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